Pricing of Fashionable Products with Strategic Consumers and Resale Marketplace

Mike Mingcheng Wei
School of Management, University at Buffalo. mcwei@buffalo.edu

Michelle X. Wu
Institute for Data, Systems, and Society (IDSS), Massachusetts Institute of Technology (MIT). wuxiao@mit.edu

John R. Birge
Booth School of Business, University of Chicago. John.Birge@ChicagoBooth.edu

This paper studies the influence of a C2C resale marketplace on pricing decisions and revenue performance of a capacitated seller selling high-tech fashion products. Under the resale marketplaces, consumers can strategically choose when (the first period or the second period), where (from the seller or from the marketplace), and what condition (new or used) to purchase. We characterize strategic consumers' purchasing equilibrium, the equilibrium market-clearing price for the resale marketplace, and the seller's optimal pricing decisions. We demonstrate that when the seller is capacitated with limited inventory, the resale marketplace will always benefit the seller. The seller can further strengthen the benefit by designing products with superior quality, a long-lasting valuation, and through cultivating early markets. Yet, with a high initial inventory, the seller benefits from the marketplace only when the first-period market size is comparatively smaller than that of the second period. Under such a scenario, the seller is better off designing fashion-oriented products with acceptable quality and attracting more non-tech-savvy consumers who typically arrive and purchase late. When the seller is able to optimally choose her initial inventory, we observe that our findings are robust under the optimal initial inventory consideration, which actually could further magnify the benefits of the marketplace. Further, we show that the existence of the marketplace could significantly improve the social welfare, especially when the product has a superior quality or a long-lasting valuation. Finally, we show that a Buy-Back Program can influence consumers' purchasing behavior and improve the seller's revenue performance.

Key words: C2C marketplaces; resale market; fashionable products; strategic consumer behavior; game theory; dynamic pricing.

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1. Introduction

In the Internet era, original equipment manufacturers (OEMs) start facing more C2C resale marketplaces that are particularly thriving for high-tech and fashion products, which are commonly characterized by highly uncertain demands and fast product obsolescence along with uncertain consumer valuation at launch. Take the smartphone industry for example, online used smartphone markets (in the following discussion, we will use the marketplace and the resale/secondary market interchangeably), such as Ebay, Swappa, and Glyde, are highly competitive, and the iPhones typically reigns supreme. This is consistent with the market observation that iPhones generally retain higher resale values among their competitors: After being used for 6 months, an iPhone retains approximately 88% of original MSRP versus 65% for Android (Dhar 2012). The high retained resale values along with liquid online markets lead to the thriving and stability of iPhones' resale market. Accordingly, consumers, especially those who are initially uncertain about their valuations, will feel less pressure to pay a premium price to purchase new devices, anticipating that they can resell them with a close-to-initial-payment price later on the marketplace.

Under the influence of marketplaces, dynamically determining prices is particularly challenging for OEMs. A liquid resale market could support the seller to charge a premium price to consumers with uncertain valuations. Yet, the resale market could potentially create direct and indirect price competitions to the primary market and thereby restrain OEMs from raising their prices. The complexity of pricing decisions is further magnified by the capacitated environment. Particularly, capacity for these high-tech and fashion products is typically built up long before the selling season, and replenishment is extremely costly, if not impossible, in the short term. Consequently, misguided pricing decisions could result in inventory shortage and substantial revenue loss, which is common in the smartphone industry (e.g., Wang 2017). Take the toy Hatchimals for another example. Hatchimals is the single biggest-selling toy at Amazon and Walmart in any category during 2016. Facing unexpected soaring demand under a price of $69.99, retailers struggled to restock their shelves, and inevitably, consumers turned to resale markets, eBay and Kijiji, paying close to original prices.

Another challenge OEMs face in pricing decisions is increasingly sophisticated customer behavior. Due to a confluence of technology, it has become more common across all income brackets and a wide variety of goods for customers to time their purchases for possible price deals (Silverstein and Butman 2006, Paragon 2011). Although strategic customer behavior has been widely studied in the literature, it is not clear how such behavior would affect the firm under the existence of the marketplaces. Certainly, the marketplace could encourage early purchases by offering a channel to resale the product later if consumer valuation turns out to be low. Yet, the marketplace may worsen the strategic waiting since consumers can wait to buy the new products in possible sales later from the primary seller and also have a chance to purchase used products available in the marketplace at lower prices.

This paper is motivated by these changes and dynamics in the marketplace and consumer behavior for high-tech and fashion products. The literature has well documented the importance of strategic consumer behavior and the resale market, but the setting that both exist in a capacitated environment has not been exploited. In this paper, we aim to develop such an understanding by considering the dynamic pricing decisions in a capacitated environment under the influence of the resale marketplace. Specifically, we investigate the following questions: how do consumers react to the marketplace, how does this reaction affect the seller's pricing decisions and revenue performance, and how should the seller plan and design the product's fashionability and deterioration to strength or weaken the impacts of the marketplace?

To answer these questions, we develop a two-period game theoretical model to understand the impact of the resale market on the seller's revenue performance. When the initial inventory level is not large, we demonstrate that the marketplace always benefits the seller. To provide intuition, consider a capacitated environment where only a limited number of consumers will be able to obtain the products in the first period. The available used units in the marketplace will be limited (which lowers the product availability in the second period) and will be traded at a higher price (which reduces the prices difference between two periods), both of which mitigate strategic waiting behavior. In addition, the limited number of used units restrains the price competition between new and used products in the second period, whereas the high price for used units in the second period supports the seller to charge a higher first-period price. Hence, the seller will benefit from the marketplace. We observe that the seller could further benefit from the marketplace by designing a product with superior quality (e.g., resistant to deterioration) and a long-lasting valuation (e.g., a classic design that won't go out of fashion quickly) and through thoroughly cultivating early markets (e.g., better informing consumers in the first period to induce more early arrivals and purchases).

When the initial inventory level is large or the seller is uncapacitated, we show that the marketplace can benefit/hurt the seller's revenue performance, if the first-period market size is comparatively smaller/larger than the second-period market size. When the first-period market size is smaller than the second-period market size, a small number of purchases in the first period will limit the supply of used products in the second period, which curbs the negative effects of the marketplace. Yet, as expected, when more consumers arrive and purchase in the first period, the benefits of the marketplace fade away due to the intensified strategic waiting behavior and price competition. Further, contrary to the limited capacity scenario, the seller can mitigate the negative influence of the marketplace by designing fashion-oriented products (e.g., with fast designed obsolescence) with an acceptable quality (e.g., with small-to-median level of deterioration) and through focusing on non-tech-savvy consumers who typically arrive and purchase late. Furthermore, when the seller is able to optimally choose her initial inventory, we observe that our findings are robust under the optimal initial inventory decision, which actually could further magnify the benefits of the marketplace.

The existence of marketplace not only benefits the seller but also could improve consumers' surplus and social welfare. In particular, we show that for a given two-period prices, consumers will always be better off under the existence of the marketplace. Furthermore, under the seller's optimal pricing decisions, we find that the marketplace can significantly improve social welfare, especially when either the valuation discount factor or the deterioration factor is not too small. At last, we explore a Buy-Back Program that proactively limits the negative influence of the marketplace. We show that by controlling the Buy-Back price, the seller can influence consumers purchasing behavior and the equilibrium market-clearing price for used products. Furthermore, we find that even with non-trivial Buy-Back costs, the Buy-Back Program can still significantly improve the seller's revenue performance.

The paper is organized as follows. Section 2 reviews the related literature. We describe the basic model setup in Section 3. Section 4 characterizes the seller's optimal pricing decisions, consumers' purchasing decisions, and the equilibrium price for the used products in the marketplace. The influence of the marketplace is discussed in Section 5. Section 6 considers an extension by including the Buy-Back Program used by the primary seller. Conclusion and future research directions are in Section 7.

2. Literature Review

Our paper is closely related to the strategic consumer behavior. Coase (1972) argues that when selling durable products in multi-period settings without the capacity constrain, monopolists must compete with their future selves, because if strategic consumers anticipate a price drop in the future, they will postpone their purchases in response. Therefore, Stokey (1979) argues that monopolists are better off to sell products at the beginning of selling season and forgo the opportunity to perform price discrimination. Strategic consumer behavior is empirically observed and studied for various industries, such as console video games (Nair 2007) and airline tickets (Li et al. 2014). In a controlled laboratory environment, Osadchiy and Bendoly (2013) find that facing a future purchase opportunity, up to 79% of customers exhibit forward-looking behavior. Such a strategic waiting behavior can have a significant detrimental impact on firms' profitability (Cachon and Swinney 2009, Zhao et al. 2016). See Netessine and Tang (2009) for a comprehensive survey of related works. Different from this line of research, our paper focuses on the case where the monopolist has limited capacity of non-durable/fashion products.

The negative influence of strategic consumer behavior can be partially limited by the capacity constrain and fashionable products. In particular, McAfee and Wiseman (2008) shows that by committing to a low production capacity, monopolists could restore, at least partially, the power of price discrimination and improve their profitability. Similar strategies have been proposed in Su and

Zhang (2008) and Liu and van Ryzin (2008). Furthermore, fashionable product's valuation typically decreases in time due to the trend chasing behavior (Li and Zhang 2013) or planned obsolescence (Miao 2011), so strategic consumers will be compelled to purchase fashionable products earlier in the selling season for higher utilities. Yet, even with fashionable products and the capacity constrain, strategic consumer behavior remains detrimental to monopolists' profit performance, especially when these monopolists adopt the dynamic pricing strategy (Aviv and Pazgal 2008, Lai et al. 2010, Aviv and Wei 2015, Chen and Hu 2018). In this paper, we introduce the role of the resale marketplace into this line of research to understand how the existence of the resale marketplace will influence strategic consumers' purchasing behavior and the seller's optimal dynamic prices.

This paper is also related to the resale market. The economics literature focuses on the influence of used durable goods on the primary market and the secondary market without capacity constraint (see Mantena, et al. 2012 for comprehensive survey), and our work differentiates from the economics literature by considering non-durable/fashionable products, whose valuations decrease in time, and the capacity constraint. In the OM literature, there are some papers studying the secondary market with limited capacity. Huang et al. (2001) compare selling and leasing options with the existence of a resale market under deterministic demand, but the resale market is not among consumers. Oraiopoulos et al. (2012) focus on how OEMs can influence the resale market by imposing relicensing fees in a B2C setting. Lee and Whang (2002) studies the impact of the secondary market on a supply chain without strategic consumers. None of these works, however, focuses on C2C platforms. In particular, Oraiopoulos et al. (2012) consider a B2C setting in which third parties refurbish used products from the first-period customers. They identified "resale value effect" and "cannibalization effects" that also appear in our paper. However, their model focuses on how an OEM charges a relicensing fee to customers who purchased refurbished products. Further, there is no inventory constraint in their model, while inventory is the key element in our study. Su (2010) characterizes the consumer behavior equilibrium in a marketplace between consumers and speculators selling tickets. It is similar to our paper in the sense that Su (2010) also emphasizes the initial capacity constraint and strategic behavior when there is demand uncertainty. The main difference is that there is no used goods and secondary marketplace but a resale market for new products by speculators. Courty (2003) and Geng et al. (2007) develop models for ticket selling where a fixed group of consumers face valuation uncertainty, and consumers who turn out to have low valuations can later resell to consumers with high valuations. Yet, similar to Su (2010), the resale marketplaces in these two papers are for new products. Our paper deviates from these papers by considering fashionable products (i.e., valuation decreases in time) and the deterioration of used products (i.e., used products have lower valuation than new ones), since it is natural for the valuations of fashion products to decrease in time and for use goods to wear out and reduce their values as well.

3. Model Settings

Consider a seller who sells Q units of fashion or high-tech products to consumers over a two-period selling season. The seller can not replenish her inventory in the selling season either because the replenishment takes a long time comparing to the short selling horizon (see Aviv and Pazgal 2008 and Aviv et al 2018) or because the capacity is built long before the selling season and therefore too costly to adjust during the selling season (see Su 2010). At the beginning of the first period, the seller sets a price p_1. In the second period, the seller can dynamically adjust her price to p_2, in response to updated information (i.e., the realized market sizes and the leftover inventory level). The market sizes, D_1 and D_2 for the first and second periods respectively, are random variables. Each consumer is infinitesimally small and, therefore, has negligible influence on other consumers' decisions. Consumers' base valuations v are heterogeneous and uncertain in nature and follow a uniform distribution between 0 and 1. A consumer can buy the product in the first period, before learning his true base valuation; or, he can delay his purchase to the second period when his valuation uncertainty has been fully resolved (Swinney 2011) through professional reviewers (e.g., cnet.com), users/developers forums (e.g., xda-developers.com), or rating websites (e.g., tomsguide.com). If the consumer chooses to wait for the second period, then his valuation for the product will be discounted by a factor $\delta \in (0,1)$ (i.e., *the valuation discount factor*). Such a decline in valuation mirrors ever-changing consumers' pursuit of new products and trend chasing behavior, or it may simply reflects the situation in which the consumer is not among the first to obtain the product (Li and Zhang 2013).

In addition to purchasing new products directly from the seller, consumers can buy used products from the marketplace. Compared to new products offered by the seller, used products are typically subject to a reduction in their valuations by a factor $\kappa \in (0,1)$. This reduction in valuation can be driven by the simple fact that there is wear and tear for used units (e.g., the scratched screen or shortened battery life for used smartphones), or by the merely psychological feeling that the new unit has been used before (Beatty 2014). Following the literature (Hendel and Lizzeri 1999, Ghose et al. 2005), we thereby refer the factor κ as *the deterioration factor*, analogous to the degradation factor as in Rust 1986 or the durability as in He et al. 2016, to represent the residual value of used products compared to new ones. In particular, a consumer with a base valuation of v will value a new product from the seller at δv in the second period and a used product from the marketplace at $\kappa\delta v$. Note that consumers who have purchased in the first period but revealed to have a low valuation may prefer to sell their used products to the marketplace, where other consumers may be interested in purchasing those used units. Jointly, supply and demand for used products endogenously determine the equilibrium market-clearing price for used products at p_2^E.

We now summarize the timeline in our model. At the beginning of the first period, the seller determines her price p_1 for new products before random market sizes D_1 and D_2 are realized. First-period consumers time their purchases (i.e., purchase immediately or wait for the second period) to maximize their expected surplus by taking into account all other consumers' purchasing strategies. Such competitive interaction among consumers will be modeled under the Nash equilibrium concept. Between the two periods, the first-period market size D_1 is realized, and the second-period market size D_2 can be learned (Su 2010). At the beginning of the second period, the seller adjusts her price to p_2. At this point, consumers' valuations are revealed privately to themselves (Swinney 2011), and consumers will determine their optimal second-period decisions (i.e., continue holding used products, buy/sell used units, purchase from the seller, or do nothing at all). Finally, all trades, used and new products, are cleared at their respective prices p_2^E and p_2. All problem parameters and distributions are common knowledge in this game. We assume that the seller and consumers are risk neutral and aim at maximizing their expected payoff.

4. The Main Model with the Marketplace

In this section, we will present the analysis of the seller's problem with the marketplace for used products – the main model. We will start with the seller's second-period problem and solve the main model recursively. There are two separated cases: No product sold in the first period or some products sold. If no consumer purchased in the first period, then there will be no used product available in the second period. Therefore, in the second period, consumers will purchase as long as their valuations are no less than the second-period price p_2 for new products. Accordingly, the seller's second-period problem in the first case can be directly stated as follows:

$$\pi_2^{NP}(D_1, D_2, Q) = \max_{0 \le p_2 \le \delta} \left\{ p_2 \cdot \min\left\{ \left(1 - \frac{p_2}{\delta}\right)(D_1 + D_2), Q \right\} \right\}, \tag{1}$$

and the seller's optimal pricing decision can be characterized by the following proposition.

PROPOSITION 1. *When no consumer purchased in the first period, it is optimal for the seller to charge her second-period price at*

$$p_2^{NP}(D_1, D_2, Q) = \begin{cases} \delta\left(1 - \frac{Q}{D_1+D_2}\right) & , \text{ if } 0 \le Q < \frac{D_1+D_2}{2}; \\ \frac{\delta}{2} & , \text{ if } Q \ge \frac{D_1+D_2}{2}. \end{cases}$$

Its corresponding optimal revenue is

$$\pi_2^{NP}(D_1, D_2, Q) = \begin{cases} \delta Q\left(1 - \frac{Q}{D_1+D_2}\right) & , \text{ if } 0 \le Q < \frac{D_1+D_2}{2}; \\ \frac{\delta}{4}(D_1 + D_2) & , \text{ if } Q \ge \frac{D_1+D_2}{2}. \end{cases}$$

Now, we consider the second case, where some new products are sold to consumers in the first period. There are two possible scenarios. Under the first scenario, all new units are sold in the first period. Accordingly, the seller's second-period decision becomes irrelevant, and the equilibrium price for used products in the marketplace p_2^E can be characterized by the following proposition.

Proposition 2. *If all new products are sold to first-period consumers, then the seller's optimal second-period price and the equilibrium price in the marketplace are* δ *and* $\delta\kappa\left(1-\frac{Q}{D_2+D_1}\right)$*, respectively. Under these prices, consumers who purchased in the first period will hold/sell used units, if their valuations are higher/less than* $\left(1-\frac{Q}{D_2+D_1}\right)$*; other consumers will attempt to purchase used units, if their valuations are higher than* $\left(1-\frac{Q}{D_2+D_1}\right)$*.*

Proposition 2 establishes the equilibrium marketplace price when all new units are sold in the first period. We notice that the marketplace price increases in both the valuation discount factor δ and the used-product deterioration factor κ. These monotonicity results directly reside in the observation that the used products bearing a higher valuation (e.g., high δ and κ) are usually sold at a higher price in the market. In addition, we observe that the marketplace price will increase in two-period market sizes (i.e., D_1 and D_2) but decrease in the initial inventory level Q. This is quite intuitive: Increasing the comparative scarcity of the product (e.g., either decreasing Q or increasing D_1 and D_2) raises the product's market-clearing price.

Last, we need to consider the second scenario, in which some units are sold to consumers in the first period and there is leftover inventory for the seller to sell in the second period. Under this scenario, we need to sequentially determine consumers' optimal purchasing decisions (§4.1), the equilibrium marketplace price (§4.2), and the seller's optimal second-period price (§**??**).

4.1. Consumers' optimal decisions in the second period

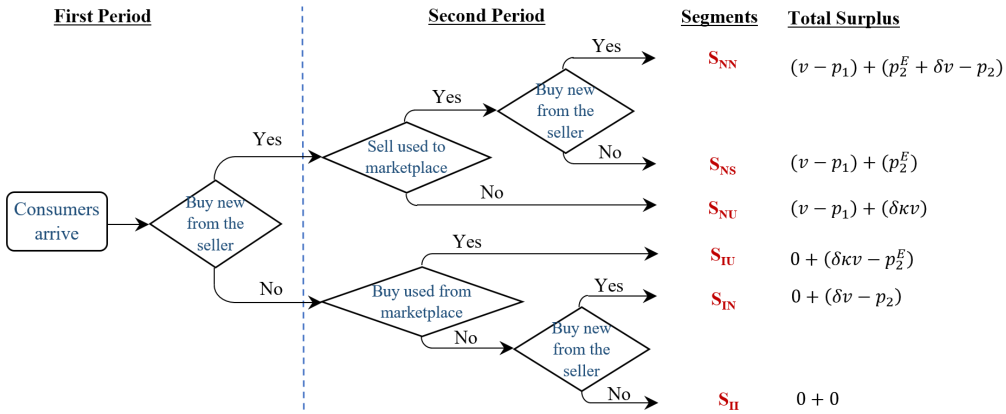
Based on possible decisions in two periods, we separate consumers into six segments: 1). S_{NN}: in this segment, consumers will buy new products from the seller in the first period, sell used units and purchase new units in the second period[1]; 2). S_{NU}: buy new products in the first period and hold used products in the second period; 3). S_{NS}: buy new products in the first period and sell used units in the second period; 4). S_{IN}: inactive/wait in the first period and buy new products in the second period; 5). S_{IU}: inactive in the first period and purchase used products from the marketplace in the second period 6). S_{II}: inactive in both periods. Figure 1 summarizes these six segments.

First consider the optimal second-period decisions for consumers who have already purchased new products in the first period. These consumers have three choices (i.e., as in S_{NN}, S_{NU}, and S_{NS}) in the second period, and their optimal decisions can be characterized in the following proposition.

Proposition 3. *For consumers who purchased products in the first period, given any arbitrary second-period price for new products* p_2 *and marketplace price for used products* p_2^E*, they will adopt the following decision rules in the second period:*

[1] We also considered a model setting, under which the S_{NN} segment does not exist and the remaining five other segments remain the same, and we found that all our analysis and insights remain qualitatively unchanged. Therefore, we merely present the main model in the paper to avoid the duplication.

Figure 1 **Consumer segmentation and decisions in two periods.**



a. *When* $p_2 > p_2^E/\kappa$*: it is optimal for consumers to act according to segment* S_{NN}*,* S_{NU}*, or* S_{NS}*, if their base valuations satisfy* $v \geq \frac{p_2 - p_2^E}{\delta(1-\kappa)}$*,* $\frac{p_2^E}{\kappa\delta} \leq v < \frac{p_2 - p_2^E}{\delta(1-\kappa)}$*, or* $v < \frac{p_2^E}{\kappa\delta}$ *respectively.*

b. *When* $p_2 \leq p_2^E/\kappa$*: it is optimal for consumers to act according to segment* S_{NN} *or* S_{NS}*, if their base valuations satisfy* $v \geq \frac{p_2}{\delta}$ *or* $v < \frac{p_2}{\delta}$ *respectively.*

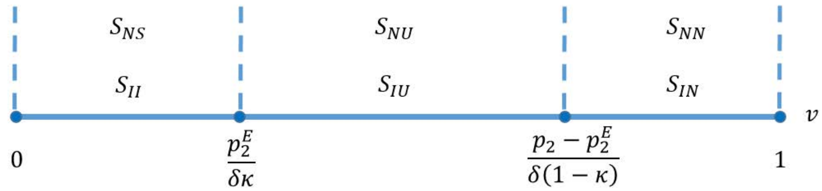
It is worth noting that Proposition 3b, the second part of this proposition, describes consumers' optimal decisions when $p_2 \leq p_2^E/\kappa$, which condition will never emerge from the equilibrium (see Theorem 1 in next subsection §4.2). Therefore, we will only discuss the first part of proposition 3.

In a nutshell, Proposition 3a demonstrates that consumers' optimal second-period decisions are contingent on their base valuations (illustrated in Figure 2). Specifically, consumers with high base valuations, $v \geq \frac{p_2 - p_2^E}{\delta(1-\kappa)}$, will attempt to sell their used products in the marketplace and purchase new units from the seller (i.e., as in segment S_{NN}). Their incentives for replacing used units with new ones come from new products' additional utilities (i.e., $\delta(1-\kappa)v$), which increase proportionally as the base valuations increase. Further, the decision of replacing used units with new ones becomes more attractive, when the price difference between the new and used units, $(p_2 - p_2^E)$, is small or when the used products will significantly impair the experience of the product (i.e., small κ). Consumers with medium base valuations, $\frac{p_2^E}{\kappa\delta} \leq v < \frac{p_2 - p_2^E}{\delta(1-\kappa)}$, may find it beneficial to continue holding their used products (as in segment S_{NU}). At last, consumers with low base valuations, $v < \frac{p_2^E}{\kappa\delta}$, just lost their interests after trying these products and therefore prefer to sell their used products in the marketplace to partially recover the price they paid before (as in segment S_{NS}).

Similarly, consumers arrived in the second period also have three possible decisions: purchase a new unit (S_{IN}), purchase a used product (S_{IU}), or do not purchase (S_{II}). The following proposition establishes these consumers' optimal decisions.

Proposition 4. *For consumers arrived in the second period, given any arbitrary new products price* p_2 *and marketplace price for the used units* p_2^E*, they will adopt the following decision rules:*

Figure 2 **Consumers' optimal decisions in the second period under the marketplace.**



1. *If* $p_2 > p_2^E/\kappa$*: it is optimal for consumers to act according to segment* S_{IN}*,* S_{IU}*, or* S_{II}*, if their base valuations satisfy* $v \geq \frac{p_2 - p_2^E}{\delta(1-\kappa)}$*,* $\frac{p_2^E}{\kappa\delta} \leq v < \frac{p_2 - p_2^E}{\delta(1-\kappa)}$*, or* $v < \frac{p_2^E}{\kappa\delta}$ *respectively.*
2. *If* $p_2 \leq p_2^E/\kappa$*: it is optimal for consumers to act according to segment* S_{IN} *or* S_{II}*, if their base valuations satisfy* $v \geq \frac{p_2}{\delta}$ *or* $v < \frac{p_2}{\delta}$ *respectively.*

A graphic illustration of this proposition is also provided in Figure 2. Directly, we observe that there is a perfect symmetry in the optimal second-period decisions between consumers who purchased in the first period (Proposition 3) and consumers who arrived in the second period (Proposition 4). This observation is not surprising. Take consumers in segment S_{NN} and S_{IN} for example. If a consumer decides to replace his used product by a new unit in the second period (S_{NN}), then it must be true that this new unit will bring a higher surplus to this consumer than either holding the used product or selling the used product. Therefore, if this same consumer arrives in the second period and did not purchase the product before, then buying a new product must be the optimal decision for him as well (S_{IN}). In other words, consumers' preferences remain the same, regardless of their purchasing history.

This unique symmetry exists only when there is a marketplace. We can show that without the marketplace, this symmetry does not hold anymore (see Figure EC.1 in Online Appendix for the case without the marketplace).

4.2. The equilibrium price in the marketplace

In the marketplace, supply of used products depends on the sizes of segments S_{NN} and S_{NS}, and demand for used products is contingent on the size of S_{IU}. The equilibrium market-clearing price is settled to match supply with demand.

Theorem 1. *For a given second-period price for new products* p_2*, the equilibrium market-clearing price for the used products is* $p_2^E = \left(\kappa p_2 - \frac{D_1}{D_1+D_2}\delta\left(1-\kappa\right)\kappa\right)^+$*.*

Theorem 1 first demonstrates that the marketplace price will be less than the new product's price adjusted by the deterioration factor (i.e., $p_2^E < \kappa p_2$), which directly suggests that the second case established in Proposition 3 and Proposition 4 will never emerge from the equilibrium.

Similar to Proposition 2, the marketplace price continues to increase in the deterioration factor κ. Yet, contrary to Proposition 2, a less expected result is that the marketplace price actually decreases in the discount factor δ. Intuitively, increasing δ improves the valuation of used products in the second period and therefore appears to suggest a higher price for used products. To explain this seemingly counter-intuitive result, note that an increase in the discount factor improves the perceived valuation of both used and new products in the second period, but in a disproportionate fashion, where the improvement is more significant for new products (as the increment for used products is further discounted by κ). In other words, as δ increases, new products become increasingly preferable over used products. Therefore, the marketplace price for used units decreases.

4.3. The seller's optimal second-period price

Next, we turn to the seller's second-period pricing decision. First, the following proposition establishes the upper and lower bounds for the optimal second-period price.

PROPOSITION 5. *The optimal second-period price resides in the domain* $\Omega = \{p_2 | p_2^{LB} \leq p_2 \leq p_2^{UB}\}$, *where* $p_2^{UB} = \delta\left(1 - \frac{D_1}{D_1+D_2}\kappa\right)$ *and*

$$p_2^{LB} = \begin{cases} \delta\left(1 - \frac{q}{D_1+D_2} - \frac{D_1}{D_1+D_2}\kappa\right), & \text{if } 0 < q \leq D_2; \\ \delta(1-\kappa)\left(1 - \frac{q}{D_1+D_2}\right)^+, & \text{if } q > D_2. \end{cases}$$

In the price domain established by Proposition 5, the seller will never induce a demand for new products that is higher than the leftover inventory level q. Therefore, we can present the seller's second-period problem as follows:

$$\pi_2(D_1, D_2, q) \doteq \max_{p_2 \in \Omega} \left\{ p_2 \cdot \left(1 - \frac{p_2 - p_2^E}{\delta(1-\kappa)}\right)(D_1 + D_2) \right\}, \tag{2}$$

where p_2^E and Ω are as defined in Theorem 1 and Proposition 5.

Solving Equation (2) is complicated by the inter-dependence between the marketplace price for used products p_2^E and the price for new products p_2. With the marketplace, changing the price for the new products will have a direct impact on the marketplace price for used units (see Theorem 1). This impact is further complicated by the interconnection among the first-period market size (which determines supply for used products), the second-period market size (which influences demand for used products), and the deterioration factor (which affects the value of used products). Therefore, the structure and presentation of the optimal second-period price are quite complex and tedious. For the interest of space, in the following theorem we only present the optimal second-period price for the case where the second-period market size is in the median range, and the analysis and solutions for all other cases are similar and can be found in the Appendix.

Theorem 2. *In the second period, if the realized second-period market size is in the median range, $D_1(1-\kappa) < D_2 \le D_1\sqrt{1-\kappa}$, then it is optimal for the seller to charge the optimal price at*

$$p_2^*(D_1, D_2, q) = \begin{cases} \delta\left(1 - \frac{q}{D_1+D_2} - \frac{D_1}{D_1+D_2}\kappa\right) & , \text{ if } 0 < q \le \frac{(1-\kappa)D_1+D_2}{2}; \\ \frac{\delta}{2}\left(1 - \frac{D_1}{D_1+D_2}\kappa\right) & , \text{ if } \frac{(1-\kappa)D_1+D_2}{2} < q \le \bar{q}; \\ \delta(1-\kappa)\left(1 - \frac{q}{D_1+D_2}\right) & , \text{ if } \bar{q} < q \le \frac{1}{2}(D_1+D_2); \\ \frac{\delta}{2}(1-\kappa) & , \text{ if } q > \frac{D_1+D_2}{2}; \end{cases}$$

where $\bar{q} = \max\left\{\frac{1}{2}(D_1+D_2)\left(1 - \sqrt{\left(1 - \frac{1}{(1-\kappa)}\left(1 - \frac{D_1}{D_1+D_2}\kappa\right)^2\right)}\right), D_2\right\}$. *The optimal second-period price $p_2^*(D_1, D_2, q)$ increases in δ and decreases in κ.*

We observe that exactly to the opposite of the case for the marketplace price p_2^E, the optimal second-period price for new units $p_2^*(D_1, D_2, q)$ increases in δ and decreases in κ. The former observation is quite intuitive. As δ increases, consumers' valuations for new products increase proportionally faster than these for used products (as explained in §4.2). Therefore, the seller is able to charge a higher second-period price. The later observation is driven by the intensified competition between new and used products in the second period. Specifically, a higher κ value increases consumers' valuations for used products and therefore homogenizes the used and new products. Accordingly, the seller will need to reduce her price for new products to attract consumers.

4.4. The first-period analysis

We are now ready to go back to the first period in which the seller optimally determines her price p_1 and consumers correspondingly choose their purchasing decisions. We start with consumers' purchasing decisions. Consider an arbitrary consumer who arrives at the first period. Define $S_2^P(v)$ as the consumer's *incremental* utility in the second period if he purchased a unit in the first period. According to Proposition 2 and Proposition 3, $S_2^P(v)$ can be written as

$$S_2^P(v) \doteq \begin{cases} (\delta v - \kappa\delta v) - \left(p_2^*(D_1, D_2, q) - p_2^E\right) & , \text{ if } D_1 < Q \text{ and } v \ge \frac{p_2^*(D_1,D_2,q) - p_2^E}{\delta(1-\kappa)}; \\ p_2^E - \kappa\delta v & , \text{ if } D_1 < Q \text{ and } v < \frac{p_2^E}{\kappa\delta}; \\ \delta\kappa\left(1 - \frac{Q}{D_2+D_1}\right) - \kappa\delta v & , \text{ if } D_1 \ge Q \text{ and } v < \left(1 - \frac{Q}{D_2+D_1}\right); \\ 0 & , \text{ otherwise.} \end{cases} \tag{3}$$

where p_2^E and $p_2^*(D_1, D_2, q)$ are determined as in Theorem 1 and Theorem 2. Utilizing this $S_2^P(v)$, we can state the expected surplus from an immediate purchase for the consumer as

$$S_P^{MP}(p_1) \doteq \mathrm{E}\left[\begin{array}{c} \min\left\{1, \frac{Q}{D_1}\right\} \cdot \left(v - p_1 + \delta\kappa v + S_2^P(v)\right) \\ + \left(1 - \min\left\{1, \frac{Q}{D_1}\right\}\right) \cdot \left(\delta\kappa v - \delta\kappa\left(1 - \frac{Q}{D_2+D_1}\right)\right)^+ \end{array}\right]. \tag{4}$$

The first part of the right-hand-side of Equation (4) calculates this consumer's surplus when he is able to obtain a unit in the first period; the second part of this equation sums up his surplus when all units are sold out and he is not able to get this product in the first period.

Similarly, the expected surplus from a wait decision for the consumer is

$$S_W^{MP} \doteq \mathrm{E}\left[\begin{array}{c} A\left(D_1 \geq Q\right) \cdot \left(\delta\kappa v - \delta\kappa\left(1 - \frac{Q}{D_2 + D_1}\right)\right)^+ \\ +A\left(D_1 < Q\right) \cdot \max\left\{\left(\delta\kappa v - p_2^E\right)^+, \left(\delta v - p_2^*\left(D_1, D_2, q\right)\right)^+\right\} \end{array}\right], \tag{5}$$

where $A(\cdot)$ is a standard indicator function. When all units are sold out in the first period, the consumer could only purchase a used product from the marketplace, i.e., the first part of Equation (5). On the other hand, when there is inventory left from the first period, he can choose between a new unit from the seller or a used product from the marketplace in the second period, i.e., the second part of Equation (5). Note that given the specification of the information structure in our model, the assessment of the decision rules adopted by all consumers in the market must be the same from each consumer's perspective. Therefore, in a pure strategy equilibrium, if it is profitable for this arbitrary consumer to purchase, then other consumers will behave the same (see Su and Zhang 2008, 2009).

The seller's first-period problem is to optimally select a price to maximize its expected two-period revenue performance, and her optimal revenue performance under the influence of the marketplace can be characterized in the following Proposition.

PROPOSITION 6. *The seller's optimal revenue performance under the marketplace $\pi^{MP}(Q)$ is given by*

$$\pi^{MP}(Q) \doteq \max\left\{\pi_{AP}^{MP}(Q), E\left[\pi_2^{NP}\left(D_1, D_2, Q\right)\right]\right\}, \tag{6}$$

where $\pi_{AP}^{MP}(Q)$ is the seller's revenue performance when some or all new products are sold to consumers in the first period:

$$\pi_{AP}^{MP}(Q) \doteq E\left[p_1^{MP} \cdot \min\left\{D_1, Q\right\} + \pi_2\left(D_1, D_2, \left(Q - D_1\right)^+\right)\right], \tag{7}$$

where

$$p_1^{MP}(Q) \doteq \frac{E\left[\min\left\{1, \frac{Q}{D_1}\right\} \cdot \left(v + \delta\kappa v + S_2^P(v)\right) + \left(1 - \min\left\{1, \frac{Q}{D_1}\right\}\right) \cdot \left(\delta\kappa v - \delta\kappa\left(1 - \frac{Q}{D_2 + D_1}\right)\right)^+\right] - S_W^{MP}}{E\left[\min\left\{1, \frac{Q}{D_1}\right\}\right]}. \tag{8}$$

Proposition 6 suggests that the seller's optimal revenue is given by the maximum between two options: The revenue generated by holding a high first-period price not to sell in the first period $\mathrm{E}[\pi_2^{NP}(D_1, D_2, Q)]$ and the revenue generated by lowering the first-period price to sell to first-period consumers $\pi_{AP}^{MP}(Q)$. It is worth noting that the second option does not necessarily dominate the first. This is because lowering the first-period price could encourage immediate purchases, but at the cost of the intensified price competition in the second period: The existence of the marketplace facilitates consumers to trade used units, which directly/indirectly compete with the seller's new products in the second/first period, and in turn hurts the seller's revenue performance.

5. The Influences of the Marketplace

In this section, we discuss the influences of the marketplace by comparing the main model analyzed in Section 4 to a benchmark model without the marketplace. As the analysis of the benchmark model is similar to the main model, we will move the technical analysis to Online Appendix. §5.1 starts with comparing the seller's second-period revenue performance; §5.2 compare the seller's total revenue in both periods when the initial inventory level is low (§5.2.1) and when the initial inventory level is high (§5.2.2); finally, the impact of the marketplace on the total social welfare is discussed in §5.3.

5.1. The seller's second-period revenue performance

In the second period, the marketplace has two counteracting effects on the seller's revenue performance. On the one hand, the marketplace enables a direct competition between used products and new products in the second period, and therefore the seller's second-period revenue performance suffers. On the other hand, high-value consumers may find it enticing to replace their used units with new ones. Conveniently, the marketplace facilitate these high-value consumers to obtain additional income by selling their used units and purchasing new devices. This positive effect may increase the second-period revenue due to the existence of the marketplace.

We, however, find that under the same leftover inventory level, the negative competition effect will always dominate, and in general the marketplace hurts the seller's second-period revenue performance. The following proposition formally establishes those observations.

Proposition 7. *Under the same leftover inventory q and given the two-period market sizes D_1 and D_2:*

a. Using the same second-period price, the seller will generate more demand for new products in the second period without the marketplace than with the marketplace.

b. The optimal second-period price is higher for the seller without the marketplace than that with the marketplace if the leftover inventory is small (e.g., $q < \min\{\kappa D_2, D_2/2\}$) or the leftover inventory is large (e.g., $q > \max\{\kappa D_2, (D_1 + D_2)/2\}$).

c. Denote $\pi_2^{NM}(D_1, D_2, q)$ as the seller's optimal second-period revenue without the marketplace. Then $\pi_2(D_1, D_2, q) \leq \pi_2^{NM}(D_1, D_2, q)$.

It is worth mentioning that although the marketplace will divert part of the demand to used products in the second period (Proposition 7a), the seller does not necessarily charge a lower second-period price (Proposition 7b). This is because the additional demand generated by the marketplace is less sensitive to price changes, when the leftover inventory is at median levels. Therefore, the seller could charge a higher price without losing too many consumers under the marketplace.

5.2. The seller's overall revenue performance

As we have shown that the marketplace generally hurts the seller's second-period revenue performance under the same leftover inventory level, the potential benefits of the marketplace will be contingent on the promise that the marketplace will support the seller selling at a higher price to more consumers in the first period. Note that the consumer who purchased the product early and revealed to have a low valuation later on will have an opportunity to partially recover his loss by reselling in the marketplace. So, the ability to sell used products in the marketplace protects consumers from the potential loss of their valuation uncertainty. Such an uncertainty-mitigation benefit of the marketplace improves the consumer's surplus from an immediate purchase decision (e.g., see $S_2^P(v)$ in Equation 3), and the consumer will be less reluctant to buy the product from the seller at a premium price.

Yet, the marketplace could potentially be detrimental to the primary seller: Offering the option for consumers to trade used products in the second period, the marketplace could provide an alternative supply source for consumers. This additional supply from the marketplace curbs the seller's ability to maintain a high price in the second period. In addition, used products will be offered at lower prices in the second period, which creates an incentive for strategic consumers to delay their purchases. Accordingly, the seller has to reduce her first-period price to discourage strategic waiting behavior and attract consumers to purchase early in the first period. Furthermore, the existence of the marketplace creates price competition: A direct competition with the primary market in the second period (e.g., consumers in the second period may prefer to buy used units) and an indirect competition with the primary market in the first period (e.g., consumers in the first period may prefer to purchase used products later). These two competitions limit the seller's ability to raise her first-period price. Together, these two interwoven negative effects argue against the marketplace.

To understand the joint influences of these effects, we will first analytically derive some managerial insights when the seller is capacitated (i.e., with limited inventory) or uncapacitated (i.e., with large inventory) in §5.2.1 and §5.2.2, respectively. Then, we will complement our findings with numerical studies to quantify the revenue impacts of the marketplace. In particular, our numerical study spans over (i) 9 levels of deterioration factor for the used products $\kappa \in \{0.1, 0.2, ..., 0.9\}$; (ii) 9 levels of consumer's valuation discount factor $\delta \in \{0.1, 0.2, ..., 0.9\}$; (iii) 7 levels of initial inventory $Q \in \{1, 5, 10, 15, ..., 30\}$; (iv) 10 levels of the average market sizes in the first period $\mathrm{E}[D_1] \in \{2, 4, ..., 20\}$; (iv) 2 correlation coefficients between two-period market sizes $\rho \in \{-1, 1\}$. Without loss of scope, we hold the mean of two periods market size at 22 (i.e., $\mathrm{E}[D_1 + D_2] = 22$) and allow the realized market sizes in each period to take two values, with equal probability and 2 units from their mean values. There are a total of $11,340$ parameter combinations that cover a wide range of practical scenarios. Across all scenarios in our numerical study, the percentage revenue improvement under the marketplace (i.e., $\pi^{MP}(Q)/\pi^{NM}(Q)-1$, where $\pi^{NM}(Q)$ is the seller's revenue without the marketplace) ranges from -33.79% to 40.68%.

5.2.1. Revenue Comparison under Limited Inventory When the seller is capacitated or the initial inventory level is not large, we find that the marketplace always benefits the seller.

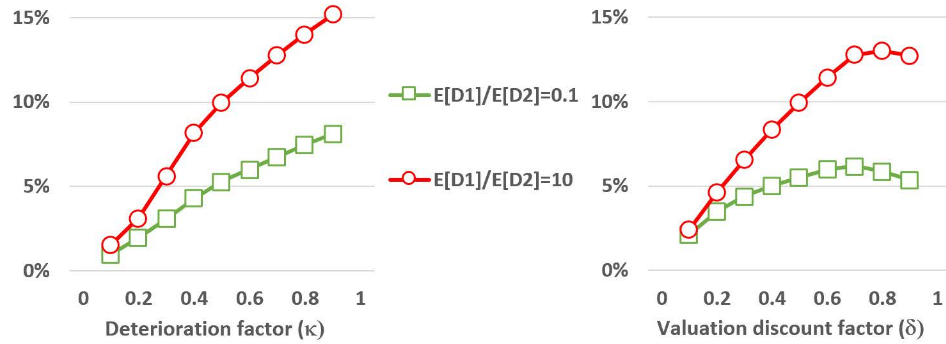
THEOREM 3. *There exists a positive Q^{LB} such that if $Q<Q^{LB}$, then $\pi^{MP}(Q)\geq\pi^{NM}(Q)$. Further, the revenue difference $\pi_{AP}^{MP}(Q)-\pi_{AP}^{NM}(Q)$ increases in δ and κ.*

We contribute this finding to two reasons. First, under the marketplace, the limited initial inventory suggests that not only may there be no new products left in the second period, but also used units in the marketplace will be sold at very high prices (see Proposition 2). In other words, the limited initial inventory could restrain strategic waiting behavior by curbing the product availability and reducing two-period price differences. Second, as the seller could sell out her inventory in the first period or be left with a small amount for the second period, the price competition between new and used products will be less severe. Combining these two reasons, we find that the limited inventory can curb both negative effects of the marketplace, and therefore the positive uncertainty-mitigation benefit dominates.

In addition, Theorem 3 shows that the seller's revenue difference with and without the marketplace, $\pi_{AP}^{MP}(Q)-\pi_{AP}^{NM}(Q)$, increases in the valuation discount factor δ and the deterioration factor κ. To gain intuition, note that as κ increases, consumers will be able to sell their used units back to the marketplace for a higher price if their valuations for the products are revealed to be low (see Theorem 1), which mitigates consumers' risk associated with their valuation uncertainty. Accordingly, the seller can benefit from charging a higher first-period price.

Gauging from a different angle, instead of the absolute revenue improvement as in Theorem 3, Figure 3 plots the average percentage revenue improvement of the marketplace, $\pi_{AP}^{MP}(Q)/\pi_{AP}^{NM}(Q)-1$, and we observe that the benefits of the marketplace also increase in κ. Although Theorem 3 shows that the revenue difference increases in δ, the percentage revenue improvement exhibits a non-monotonic pattern (see the right plot of Figure 3). Specifically, the marketplace seems to benefit the seller the most when δ is in the median to high range. At last, Figure 3 also illustrates the impacts of the two-period market size ratio. We observe that benefits of the marketplace seems to increase in the ratio of the average market sizes in two periods, $\mathrm{E}[D_1]/\mathrm{E}[D_2]$. The improved benefits of the marketplace can be attributed to increasing likelihood of the seller to sell out her inventory in the first period or to be left with limited units in the second period and therefore dilute the aforementioned negative effects of the marketplace.

Figure 3 **The benefits of the marketplace, $\pi^{MP}(Q)/\pi^{NM}(Q)-1$, when $Q=5$. Note: Theorem 3 uses the measure of the revenue difference, and this figure uses the revenue improvement.**



In sum, in industries where the seller typically maintains limited initial inventory or capacity, the seller should support and encourage consumers to trade used products in the marketplace. Moreover, the seller could benefit from the marketplace further through designing a product with superior quality (e.g., resistant to deterioration, i.e., a large κ) and a long-lasting valuation (e.g., won't go out of fashion quickly, i.e., a median-to-large δ) and through thoroughly cultivating early markets (e.g., better informing consumers, focus on tech-savvy consumers, and expand the market in early stage, i.e., not too small $\mathrm{E}[D_1]/\mathrm{E}[D_2]$). In the smartphone example, comparing to Android-based phones, iPhone has comparatively much smaller production capacity and follows similar strategies to benefit from the marketplace, such as honoring transferable product warranty, producing high quality products, introducing new products at a slow pace, targeting enthusiastic fans who are well informed and eager to purchase early. As a result, we observe that in the marketplace, iPhone typically maintains a higher resale value, which in turn supports Apple to charge a premium first-period price and benefit from the marketplace.

5.2.2. Revenue Comparison under Large Inventory When the seller is uncapacitated or the initial inventory is large, however, we observe that the marketplace only benefits the seller when the two-period market size ratio is not large. In particular, we have the following theorem.

THEOREM 4. *There exists positive constants Q^{UB}, R^{LB}, and R^{UB} such that for the market size pair D_1 and D_2 and the initial inventory $Q > Q^{UB}$:*

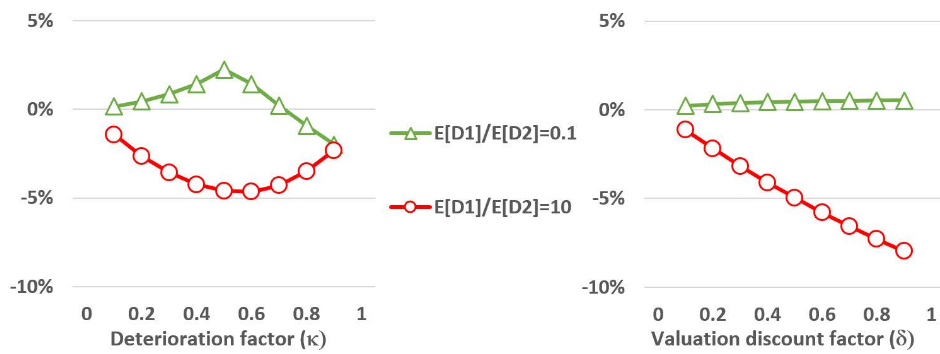
(a) if $D_1/D_2 < R^{LB}$, then $\pi^{MP}(Q) \geq \pi^{NM}(Q)$. Further, $\pi^{MP}(Q) - \pi^{NM}(Q)$ increases in δ, increases in κ for $\kappa \leq \frac{1}{2}$, but decreases in κ for $\kappa > \frac{1}{2}$.

b. If $D_1/D_2 > R^{UB}$, then $\pi^{MP}(Q) \leq \pi^{NM}(Q)$. Further, $\pi^{MP}(Q) - \pi^{NM}(Q)$ decreases in δ, decreases in κ for $\kappa \leq \frac{1}{2}$, but increases in κ for $\kappa > \frac{1}{2}$.

Theorem 4 demonstrates that under a high initial inventory, the marketplace will benefit the seller, only when the first-period market size is comparatively smaller than the second-period market size (Theorem 4a). This is because a small number of purchases in the first period will limit the supply of used products in the second period and hence curb the negative effects of the marketplace. Yet, when more consumers arrive and buy in the first period, the benefits of the marketplace fade away (Theorem 4b).

Using the parameter combinations described in Section 5.2, Figure 4 plots the average revenue improvement, $\pi^{MP}(Q)/\pi^{NM}(Q)-1$, for a large initial inventory level (e.g., $Q=30$). Theorem 4

Figure 4 **The benefits of the marketplace, $\pi^{MP}(Q)/\pi^{NM}(Q)-1$, when $Q=30$. Note: Theorem 4 uses the measure of the revenue difference, and this figure uses the revenue improvement.**



also suggests that the influences of the deterioration rate κ and the valuation discount factor δ depend on the two-period market size ratio. The deterioration factor κ has two conflicting effects. On the one hand, used products are more valuable for a large κ, and therefore consumers are able to sell their used products in the marketplace for a higher price (see Theorem 1), which supports a higher first-period price. On the other hand, a high deterioration factor κ will assimilate the difference between used and new products, which forces the seller to drop her second-period price (see Theorem 2). Therefore, the seller could suffer from intensified price competition and strategic consumer behavior under a large κ. When the market size ratio is small (i.e., less consumers arrive in the first period), there will be limited amount of used products available in the marketplace, which curbs the marketplace's negative effects. Hence, the marketplace generally benefits the seller. Such benefit first mildly increases in κ (as the seller can charge a higher first-period price to a small number of consumers in the first-period) and then sharply decreases (as the seller suffers from the intensified price competition in the second period).

However, when the market size ratio is large, there are sufficient used products available in the marketplace to compete with the seller. The marketplace generally hurts the seller's revenue performance, due to the intensified the price competition and strategic consumer behavior. And similarly, increasing κ will both intensify the price competition in the second period (because of reducing the valuation difference between used and new products) and support a higher first-period price (because of the higher marketplace price). Yet, just to the opposite of the small market size ratio scenario, the benefits of the marketplace under the large market size ratio first decrease and then increase in κ. This is because there are a large number of consumers in the first period, and the benefits of supporting a higher first-period price will be enhanced under a higher κ.

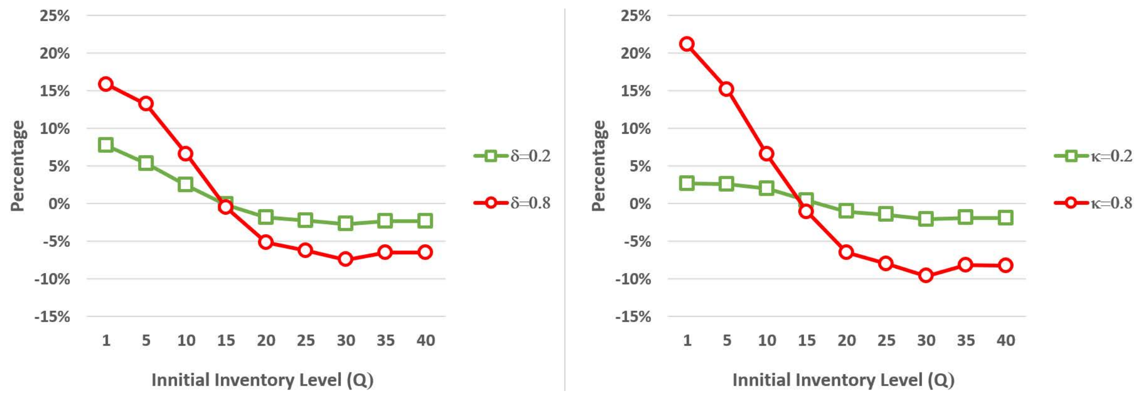
Likewise, the impact of the valuation discount factor δ on the benefits of the marketplace also depends on the market size ratio. When the market size ratio is large (more consumers arrive in the first period), more available used products in the second period will intensify the negative effects of the marketplace. In addition, recall from Theorem 1 that the increasing δ will decrease the market-clearing price for used products, which limits the seller's ability to charge a higher first-period price. Accordingly, the benefits of the marketplace decrease in δ. On the other hand, when the market size ratio is small, the negative influences of the marketplace will be limited by the insufficient number of used products. Therefore, we observe that the benefits of the marketplace mildly increase in δ.

In sum, in industries where high capacity/inventory is a standard practice (due to low production cost or competition, for example), the seller is better off avoiding the marketplace or at least limiting its influence. In the smartphone example, Android-based smartphone has much high production quantity and potentially suffers from the marketplace. Hence, the seller attempts to mitigate the negative influence of the marketplace by discouraging marketplace trading (e.g., voiding product warranty if purchased from the marketplace, see LG 2013). In addition, we suggest that the seller could tactically limit the negative influence of the marketplace by designing fashion-oriented products (e.g., releasing new products more frequently and having fast designed obsolescence , i.e., small δ) with an acceptable quality (i.e., small to median level of κ) and through focusing non-tech-savvy consumers who typically arrive and purchase late (e.g., small $\mathrm{E}[D_1]/\mathrm{E}[D_2]$).

5.2.3. The Impact of Initial Inventory Now, we will discuss the impact of the initial inventory level. When the initial inventory is given, Figure 5 plots the average revenue improvement, $\pi^{MP}(Q)/\pi^{NM}(Q)-1$, for different initial inventory levels.

As explained before, the marketplace generally benefits/hurts the seller's revenue performance under a low/high initial inventory. Further, we observe that the benefits of the marketplace tends to first decrease and then increase in the initial inventory level Q. As discussed earlier, when the initial inventory level is not too large, the leftover inventory for the second period will be limited, which curbs the negative influences of the marketplace. Accordingly, the seller will benefit from the positive uncertainty-mitigation effect. As the initial inventory increases, the seller will need to compete with increasing amount of used products from the marketplace and suffer from intensified strategic waiting behavior. Hence, the benefits of the marketplace fade away. On the other side of

Figure 5 **The impact of the initial inventory level, Q, on the benefits of the marketplace, $\pi^{MP}(Q)/\pi^{NM}(Q)-1$.**



spectrum, when the seller has a large amount of initial inventory, her revenue performance can be negatively influenced by the marketplace due to strategic consumer behavior and price competitions. Yet, the seller is better off to have a large amount of inventory, under which the seller may compensate themselves by serving more consumers, than to have a median level of inventory, under which the seller needs to directly compete with the marketplace for a few consumers. Therefore, when the initial inventory level is in the median range, the marketplace hurts the seller the most.

Next, we consider the scenario in which the seller can choose the optimal level of inventory Q. We adopt a standard approach: consider a given per-unit ordering/production cost, c, the seller attempts to optimize her expected profit by identifying the inventory level and best pricing strategy. For example, under the model with the marketplace, we solve the following problem: $\Pi^{MP} \doteq \max_Q \{\pi^{MP}(Q) - cQ\}$. Similarly, we calculate the optimal expected profit for the no marketplace case (denoted as Π^{NM}). Utilizing the same parameter combinations, we searched for the optimal profit for the 5 ordering/production costs: $c \in \{0.1, 0.2, ..., 0.5\}$. For example, Table 1 below summarized 4 scenarios, spanned by the combinations of $\delta = 0.5$, $\kappa = 0.5$, $\mathrm{E}[D_1] + \mathrm{E}[D_2] = 30$, $\rho = 1$, two cost parameter values: $c = 0.2$ and 0.4, and two market size ratio ($\mathrm{E}[D_1]/\mathrm{E}[D_2]$) values: 0.5 and 2.

Table 1 The impact of the optimal inventory decision on the benefits of the marketplace for the case $\delta = 0.5$, $\kappa = 0.5$, $\mathrm{E}[D_1] + \mathrm{E}[D_2] = 30$, and $\rho = 1$.

			E[D$_1$]/E[D$_2$]=0.5				E[D$_1$]/E[D$_2$]=2			
			Q*	p$_1$	Profit (Π)	Benefits	Q*	p$_1$	Profit (Π)	Benefits
c	0.2	Marketplace	15.00	0.62	4.13	9.38%	25.00	0.55	6.56	-7.43%
		No Marketplace	20.00	0.58	3.78		27.81	0.57	7.09	
	0.4	Marketplace	8.59	0.66	1.69	22.69%	15.00	0.60	2.96	14.96%
		No Marketplace	10.31	0.61	1.38		21.41	0.58	2.58	

This table first demonstrates that the marketplace could significantly influence, both positively and negatively, the seller's profitability. Moreover, the seller under the influence of the marketplace tends to understock (i.e., order less initial inventory). Recall that we have demonstrated that limiting the initial inventory could effectively mitigate the negative influence of the marketplace. Therefore, the leverage of optimally choosing the initial inventory seems to give the seller an edge and benefit the seller's profitability. Particularly, the marketplace is most beneficial, when the products are expensive to order/produce (i.e., c is large). This is exactly where the seller will order/produce limited initial inventory and benefits from the marketplace.

5.3. Social Welfare

The existence of the marketplace not only influences the seller's revenue performance, but also could directly affect each and every consumer's surplus. In this subsection, we will briefly discuss the impact of the marketplace on consumer's surplus and further seek the answer to the question of how the marketplace influences the social welfare.

We first find that for given two-period prices, consumers will not be worse off under the marketplace. Particularly, by directly comparing individual consumer's surplus with the marketplace (Proposition 3 and Proposition 4) and without the marketplace (Appendix 1), we have the following proposition.

PROPOSITION 8. *Given the first-period and second-period prices p_1 and p_2 and initial inventory level Q, individual consumer's surplus will be improved or stay the same under the marketplace:*

I. Among consumers who arrived and purchased in the first period, consumers with high valuations (in segment S_{NN}) and low valuations (in segment S_{NS}) will be better off under the marketplace, and consumers with median valuations (in segment S_{NU}) will be indifferent with or without the marketplace.

II. Among consumers who arrived in the second period, consumers with median valuations (in segment S_{IU}) will be better off under the marketplace, and consumers with high valuations (in segment S_{II}) and low valuations (in segment S_{IN}) will be indifferent with or without the marketplace.

Proposition 8 seems to allude to the conclusion that the total consumer surplus will be improved under the marketplace. Yet, the above proposition is somewhat deceptive, as it has fixed the seller's actions (e.g., pricing and inventory decisions). Therefore, it is clearly indisputable that consumers can do no worse by having the option to purchase/sell used products from/to the marketplace. However, once taking the seller's optimal decisions into consideration, it is no longer obvious that individual consumers and the society as a whole will benefit from the marketplace. Due to the complexity of our model setting, analytically comparing the total social welfare are prohibitively difficult, if not impossible. Therefore, we utilize the same parameter combinations as in §5.2 to numerically assess the impact of the marketplace on the social welfare. Our approach is standard. For example, to obtain the total consumers' surplus under the marketplace, we calculate and aggregate each and every individual consumer's surplus under the optimal two period prices (p^*_{MP} and $p^*_2(D_1, D_2, q)$) and the equilibrium marketplace price (i.e., p^E_2); and then we sum the total consumers surplus and the seller's revenue performance ($\pi^{MP}_1(Q)$) to gauge the social welfare under the marketplace (denoted by $SW^{MP}(Q)$). Similarly, we calculate the social welfare without the marketplace (denoted as $SW^{NM}(Q)$).

We find that under most scenarios (82.08% of all $13,310$ instances), the existence of the marketplace improves the social welfare (with an average improvement of 37.39%). Moreover, the improvement is quite significant when either the valuation discount factor δ or the deterioration factor κ is not too small. In addition, we observe that the social welfare tends to improve as κ or δ increases. This is not surprising. As consumers trade used products in the second-period, increasing the valuation of used products (i.e., increase κ or δ) will magnify the influence of the marketplace. Further, we observe that the marketplace could substantially improve the social welfare when the initial inventory is not too large. This is exactly what happens when the optimal inventory decision is taken into consideration.

6. Extension

Facing growing marketplaces, some major OEMs choose to encourage the resale market to gain competitive advantage over their rivals (e.g., IBM and HP, see Oraiopoulos 2012), while other OEMs adopt strategies to suppress the influence of the resale market or even eliminate the marketplace (e.g., Marion 2004). Certainly, by discouraging trades in the marketplace, the seller could shield itself from the negative influences of the marketplace, but doing so will simultaneously weaken the positive effect. In this section, we will explore a Buy-Back program that proactively directs, not eliminates, the marketplace.

The Buy-Back program facilitates consumers to sell their used products back to the seller at a discounted price $r \geq 0$. For example, consumers can sell their used units back to Apple through its Renew and Recycle Program, which is administrated by its long-time partner Brightstar. Typically, the seller determines the Buy-Back price before or immediately after the official launch of its devices (e.g., see Kharif et al 2014 for iPhone 6's example), and the majority of these Buy-Back units are either scrapped for parts or resold in other emerging markets such as Asia, South America, and eastern Europe to prevent cannibalization (Burrows 2013a 2013b and Kharif et al 2014). In other words, most of the used units sold back to Apple will not re-enter the market in the US and therefore do not affect the resale market. Further, as recycling these units could be costly or could generate additional revenue for the seller (Kim 2016), we will denote such cost/revenue as s.

As the section is for illustration purposes, we will directly summarize the influence of the Buy-Back program on consumers' behavior and the seller's revenue performance. Please refer to Online Appendix for detailed analysis. When the seller offers the Buy-Back program, consumers with used units in the second period can choose to continue holding their used units, sell in the marketplace at price $\tilde{p}_2^E$, or sell back to the seller at price r. Proposition EC.5 in the appendix demonstrates how the Buy-Back price influences the marketplace price. In particular, when the seller posts a Buy-Back price that is lower than the marketplace price (e.g., $r \leq p_2^E$), the Buy-Back program has no impact on consumers' behavior; and no consumer will choose to sell their used products back to the seller. When the Buy-Back price is higher than the marketplace price but lower than $\kappa\tilde{p}_2$, where $\tilde{p}_2$ is the second-period price for the new products, then the Buy-Back program will raise the marketplace price, which intensifies the positive uncertainty-mitigation effect. Therefore,increasing the Buy-Back price could benefit the seller. At last, when the Buy-Back price is higher than $\kappa\tilde{p}_2$, consumers will strictly prefer selling their used units back to the seller instead of trading in the marketplace, under which scenario the seller essentially eliminates the marketplace by buying back consumers' used devices. This option could be feasible, especially when the negative influences of the marketplace prevail.

We denote $\pi^{BB}(Q)$ as the optimal two-period revenue performance under the Buy-Back program, and Theorem EC.1 in the appendix demonstrates that the Buy-Back program benefits the seller (i.e., $\pi^{BB}(Q) \geq \pi^{MP}(Q)$). To quantitatively gauge the benefits of the Buy-Back program, we extend the existing numerical study by considering five values of the reselling or scrapping revenue $s \in \{-0.2, -0.1, 0, 0.1, 0.2\}$ and searching the optimal Buy-Back price r to maximize the seller's revenue performance. Across all scenarios, the percentage revenue improvement from the Buy-Back program, $\pi^{BB}(Q)/\pi^{MP}(Q) - 1$, ranges from 0% to 78.93%, with an average value of 12.82%. As expected, the benefits of adopting the Buy-Back program improve as the reselling or scrapping revenue s increases. Yet, even when $s = 0$ (e.g., the seller pays positive Buy-Back price to recycle used units from consumers and receives zero revenue from reselling and scrapping), the seller will still benefit from adopting the Buy-Back program (e.g., the average, minimum, and maximum benefits are 3.50%, 0%, and 43.10%). These benefits demonstrate that the Buy-Back Program, even with non-trivial Buy-Back costs, will significantly benefit the seller.

7. Conclusion

The importance of strategic consumer behavior and the resale market has been well documented in the literature, but the setting that both exist in a capacitated environment has not been exploited. In this paper, we aim to develop such an understanding by developing a game-theoretical model to study the influence of the online C2C resale market on the primary seller's pricing decisions and revenue performance. We demonstrate that when the initial inventory level is not large, the marketplace always benefits the seller. Such benefit can be further strengthened, if the seller designs her products with superior quality and a long-lasting valuation and cultivates her early markets. Yet, when the seller is uncapacitated or the initial inventory level is large, we show that the marketplace will benefit the seller only when the first-period market size is comparatively smaller than the second-period market size. Under this scenario, the seller is better off to design fashion-oriented products with an acceptable quality and to focus on non-tech-savvy consumers who typically arrive and purchase late. Furthermore, the existence of the marketplace not only benefits the seller but also could improve consumers' surplus and the social welfare, especially for products with a superior quality or a long-lasting valuation. We also observe that our findings are robust under the optimal initial inventory decision, which actually could further magnify the benefits of the marketplace. At last, we explore the Buy-Back program's influence on consumers' purchasing behavior and the seller's revenue performance, and show that even with non-trivial buy-back costs, the Buy-Back program can still significantly improve the seller's revenue performance.

This work can be extended in several directions. First, the asymmetric information (e.g., the product quality) between consumers and the seller could be partially resolved through observing prices for both new and used units, and therefore endogenizing this information asymmetry could unveil another potential benefit of the marketplace. Second, we do not consider the costs associated with changing the product's design, which may influence the product's cost structure. Therefore, explicitly including the product design (e.g., quality and fashionability) and her associated costs could further assist the seller in planning her long-term strategic decisions. Third, including the impact of refurbished products from third parties further extends our model from the C2C marketplace to include a B2C platform that could be an influential factor in certain industries (e.g., Oraiopoulos 2012). At last, exploring the joint influence of product updates, so that the products offered in the second period can have higher valuation than in the first period, and the Buy-Back program can assist the seller to plan and promote her new product launches.

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Appendix: Proofs

Proof of Proposition 1 We first solve Equ (1) without the inventory constrain: $p_2 \cdot \left(1-\frac{p_2}{\delta}\right)(D_1+D_2)$, which is a concave function and maximized at $p_2^* = \frac{\delta}{2}$. At $p_2^* = \frac{\delta}{2}$, the seller will induce $\frac{D_1+D_2}{2}$ units of demand. Hence, when the initial inventory is higher than $\frac{D_1+D_2}{2}$, then it is optimal for the seller to set price at $\frac{\delta}{2}$; otherwise, the seller will set price to sell all of her inventory.

Proof of Proposition 2 First, we consider consumers optimal decisions. For an given equilibrium price $0 \le p_2^E \le \delta$, a typical consumer who holds a used unit with a base valuation v will prefer to continue possessing this unit if the surplus from doing so is no less than that from selling in the marketplaces: $0 \ge p_2^E - \delta\kappa v$ or equivalently $v \ge p_2^E/\delta\kappa$. Similarly, a consumer who do not have a product on hand will prefer to purchase a used unit from the market as long as doing so improves his surplus: $\delta\kappa v - p_2^E \ge 0$ or equivalently $v \ge p_2^E/\delta\kappa$. Now, we need to derive the equilibrium price. When all units are sold to consumers in the first period, we know that the number of used units at the beginning of the second period is Q and the number of consumers who are in the market and do not have used units is $D_2 + (D_1 - Q)$, where $D_1 \ge Q$. To match supply with demand, the equation $Q\left(\frac{p_2^E}{\delta\kappa} - 0\right) = (D_2 + (D_1 - Q))\left(1 - \frac{p_2^E}{\delta\kappa}\right)$ must hold for the equilibrium price p_2^E, solving which gives us the equilibrium price $\delta\kappa\left(1 - \frac{Q}{D_2+D_1}\right)$.

Proof of Proposition 3 Denote the *increment* of surplus for segments S_{NN}, S_{NU}, and S_{NS} as SP_{NN}, SP_{NU}, and SP_{NS} respectively. As consumers in segment S_{NN} choose to replace the used units for new units, their incremental surplus is $SP_{NN} = p_2^E - \kappa\delta v + \delta v - p_2$. Similarly, we have $SP_{NU} = 0$ and $SP_{NS} = p_2^E - \kappa\delta v$. As rational consumers choose their decisions to maximize their surplus, it must be true that for consumers in segment S_{NN} must have $SP_{NN} \ge \max\{SP_{NU}, SP_{NS}\}$, which can be simplified into $v \ge \max\left\{\frac{p_2}{\delta}, \frac{p_2 - p_2^E}{\delta(1-\kappa)}\right\}$. Analogously, for a consumer to be in segment S_{NU} or S_{NS}, his base valuation must satisfies $v \in \left[\frac{p_2^E}{\kappa\delta}, \frac{p_2 - p_2^E}{\delta(1-\kappa)}\right]$ and $v \le \min\left\{\frac{p_2}{\delta}, \frac{p_2^E}{\kappa\delta}\right\}$. Next, we will need to consider the relationship between $\frac{p_2}{\delta}$ and $\frac{p_2^E}{\kappa\delta}$. If $\frac{p_2}{\delta} > \frac{p_2^E}{\kappa\delta}$, then we immediately have $\frac{p_2 - p_2^E}{\delta(1-\kappa)} > \frac{p_2}{\delta}$. Hence, consumers whose base valuations satisfy $v \ge \frac{p_2 - p_2^E}{\delta(1-\kappa)}$, $\frac{p_2^E}{\kappa\delta} \le v < \frac{p_2 - p_2^E}{\delta(1-\kappa)}$, or $v < \frac{p_2^E}{\kappa\delta}$ will belongs to segments of S_{NN}, S_{NU}, or S_{NS} respectively. On the other hand, if $\frac{p_2}{\delta} \le \frac{p_2^E}{\kappa\delta}$, then we will have $\frac{p_2 - p_2^E}{\delta(1-\kappa)} \le \frac{p_2}{\delta}$. Under this scenario, there will be no consumer belonging to segment S_{NU}, and consumers whose base valuations satisfy $v \ge \frac{p_2}{\delta}$ or $v < \frac{p_2}{\delta}$ will belongs to segments of S_{NN} or S_{NS} respectively.

Proof of Proposition 4 This proof is similar to the proof of Proposition 3 and therefore omitted.

Proof of Theorem 1 First we will show that the second case in Proposition 3 and Proposition 4 where $p_2 \le p_2^E/\kappa$ will never emerge in the equilibrium. To see, if $p_2 \le p_2^E/\kappa$, then the demand for used product is always zero, but the supply of the used product is non-negative (i.e., the sum of consumers in both S_{NN} and S_{NS} segments equals the number of consumers arrived in the first period), which will never sustain in the equilibrium. Hence, we can focus on the first case (i.e., $p_2 > p_2^E/\kappa$). We first equalize demand and supply of used products in the marketplace:

$$\left(\frac{p_2 - p_2^E}{\delta(1-\kappa)} - \frac{p_2^E}{\kappa\delta}\right) D_2 = \left(1 - \frac{p_2 - p_2^E}{\delta(1-\kappa)}\right) D_1 + \left(\frac{p_2^E}{\kappa\delta}\right) D_1,$$

solving which gives $p_2^E = \kappa p_2 - \frac{D_1}{D_1+D_2}\delta(1-\kappa)\kappa$. At last, the marketplace price must be non-negative (otherwise, the consumers are better off to dispose their used products at zero cost). Therefore, we will set the equilibrium price at zero when $p_2 < \frac{D_1}{D_1+D_2}\delta(1-\kappa)$, under which scenario besides satisfying the demand, the excessive supply of used products will be disposed at zero cost.

Proof of Proposition 5 The price upper-bound is the lowest price under which no consumer will attempt to purchase a new unit (according to Proposition 3 and Proposition 4). Therefore, any price equal or higher than this upper-bound will yield a zero revenue for the seller. There are two cases depending on the relationship between D_2 and q. If $q = D_2$, then in order to sell exactly q units of her inventory, the seller will set her second period price to $p_2 = \frac{D_1}{D_1+D_2}\delta(1-\kappa)$, under which the corresponding equilibrium market price will be $p_2^E = 0$ (see Theorem 1). Therefore, when $q > D_2$, to sell all of her inventory out in the second period, the seller needs to set a second-period price lower than $\frac{D_1}{D_1+D_2}\delta(1-\kappa)$, under which the equilibrium market price will be kept at $p_2^E = 0$ and the corresponding demand for new products is $\left(1 - \frac{p_2}{\delta(1-\kappa)}\right)(D_1 + D_2)$. If $D_2 < q \leq (D_1 + D_2)$, then the seller wil sell all of her inventory off for a non-negative price which satisfies the equation $\left(1 - \frac{p_2}{\delta(1-\kappa)}\right)(D_1 + D_2) = q$, solving which gives $p_2 = \delta(1-\kappa)\left(1 - \frac{q}{D_1+D_2}\right)$. Further note that when $q > (D_1 + D_2)$, the maximum number of new products the seller could sell is $(D_1 + D_2)$. To do so, the seller will set its price to be 0 and the remaining unsold inventory will be disposed at zero cost. In sum, when $q > D_2$, to maximize her profit performance, the seller will never set her second period price lower than $\delta(1-\kappa)\left(1 - \frac{q}{D_1+D_2}\right)^+$.

When $q \leq D_2$, the equilibrium market price will be $p_2^E = \kappa p_2 - \frac{D_1}{D_1+D_2}\delta(1-\kappa)\kappa \geq 0$ from Theorem 1. Moreover, the seller could sell all of her inventory for a positive price that satisfies the equation $\left(1 - \frac{p_2 - p_2^E}{\delta(1-\kappa)}\right)(D_1 + D_2) = q$, solving which gives us $p_2 = \delta\left(1 - \frac{q}{D_1+D_2} - \frac{D_1}{D_1+D_2}\kappa\right)$. Hence, when $q \leq D_2$, the seller will never set her second-period price lower than $\delta\left(1 - \frac{q}{D_1+D_2} - \frac{D_1}{D_1+D_2}\kappa\right)$.

Proof of Theorem 2 First, we can exclude the case where $q > D_1 + D_2$, which has the same revenue as the case where $q = D_1 + D_2$. Now, we will discuss the seller's second-period optimal pricing decisions for two cases: Case I where $0 < q \leq D_2$ and Case II where $D_2 < q \leq D_1 + D_2$.

Case I: From the proof of Proposition 5, the optimal second-period price will be no less than $p_{2,1}^{LB} \doteq \delta\left(1 - \frac{q}{D_1+D_2} - \frac{D_1}{D_1+D_2}\kappa\right)$, under which $p_2^E = \kappa p_2 - \frac{D_1}{D_1+D_2}\delta(1-\kappa)\kappa \geq 0$. We can rewrite the seller's revenue performance $\Pi_{2,1}(p_2)$ as

$$\Pi_{2,1}(p_2) = p_2 \cdot \left(1 - \frac{p_2}{\delta} - \frac{D_1}{D_1+D_2}\kappa\right)(D_1 + D_2), \tag{9}$$

which is a concave function and maximized at $p_{2,1} \doteq \frac{\delta}{2}\left(1-\frac{D_1}{D_1+D_2}\kappa\right)$. Note that the seller will sell out her inventory if the second-period price is set to $p_{2,1}^{LB}$. Therefore, when $p_{2,1} \geq p_{2,1}^{LB}$, the optimal second-period price $p_{2,1}^*$ is $p_{2,1}$; otherwise, $p_{2,1}^* = p_{2,1}^{LB}$. In other words, the optimal second-period price for the case where $0 < q \leq D_2$ can be identified as follows:

$$p_{2,1}^* = \begin{cases} \frac{\delta}{2}\left(1-\frac{D_1}{D_1+D_2}\kappa\right) & ;\ q \geq \frac{(1-\kappa)D_1+D_2}{2} \\ \delta\left(1-\frac{q}{D_1+D_2}-\frac{D_1}{D_1+D_2}\kappa\right) & ;\ q < \frac{(1-\kappa)D_1+D_2}{2} \end{cases}. \tag{10}$$

Case II: Following Proposition 5, the optimal second-period price is lower-bounded by $p_{2,2}^{LB} \doteq \delta(1-\kappa)\left(1-\frac{q}{D_1+D_2}\right)$, and the market equilibrium price p_2^E can have two different values:

$$p_2^E = \begin{cases} \kappa p_2 - \frac{D_1}{D_1+D_2}\delta(1-\kappa)\kappa & ;\ \frac{D_1}{D_1+D_2}\delta(1-\kappa) \leq p_2 \leq p_2^{UB} \\ 0 & ;\ p_{2,2}^{LB} \leq p_2 < \frac{D_1}{D_1+D_2}\delta(1-\kappa) \end{cases}.$$

When $p_2 \in \left[\frac{D_1}{D_1+D_2}\delta(1-\kappa), p_2^{UB}\right]$, the seller's revenue function is $\Pi_{2,1}(p_2)$, which is defined in Equ (9) and solved by Equ (10). When $p_2 \in \left[p_{2,2}^{LB}, \frac{D_1}{D_1+D_2}\delta(1-\kappa)\right]$, the corresponding revenue function $\Pi_{2,2}(p_2)$ can be stated as follows:

$$\Pi_{2,2}(p_2) \doteq p_2 \cdot \left(1-\frac{p_2-0}{\delta(1-\kappa)}\right)(D_1+D_2),$$

which is also a concave function and maximized at $p_{2,2} \doteq \frac{\delta}{2}(1-\kappa)$. To determine the optimal second-period price, we need to consider the following seven cases under the condition of $D_2 < q \leq D_1 + D_2$:

1. $p_{2,1}$ is an interior solution within $\left[\frac{D_1}{D_1+D_2}\delta(1-\kappa), p_2^{UB}\right]$ and $p_{2,2} \geq \frac{D_1}{D_1+D_2}\delta(1-\kappa)$: Together with the initial requirement of $D_2 < q \leq D_1 + D_2$, the conditions for this case are $D_2 < q \leq D_1 + D_2$ and $D_2 \geq D_1$, under which the profit is maximized at $p_{2,1}$.
2. $p_{2,1}$ is an interior solution within $\left[\frac{D_1}{D_1+D_2}\delta(1-\kappa), p_2^{UB}\right]$, $p_{2,2}$ is an interior solution within the range of $\left[0, \frac{D_1}{D_1+D_2}\delta(1-\kappa)\right]$, and $\Pi_{2,1}(p_{2,1}) \geq \Pi_{2,2}(p_{2,2})$: Together with the initial requirement of $D_2 < q \leq D_1 + D_2$, the conditions for this case are $D_2 \leq q \leq D_1 + D_2$ and $D_1\sqrt{1-\kappa} \leq D_2 \leq D_1$, under which profit is maximized at $p_{2,1}$.
3. $p_{2,1}$ is an interior solution within $\left[\frac{D_1}{D_1+D_2}\delta(1-\kappa), p_2^{UB}\right]$, $p_{2,2}$ is an interior solution within the range of $\left[p_2^{LB}, \frac{D_1}{D_1+D_2}\delta(1-\kappa)\right]$, and $\Pi_{2,1}(p_{2,1}) \leq \Pi_{2,2}(p_{2,2})$: Together with the initial requirement of $D_2 < q \leq D_1 + D_2$, the conditions for this case are $\frac{1}{2}(D_1+D_2) \leq q \leq D_1 + D_2$ and $D_1(1-\kappa) \leq D_2 \leq D_1\sqrt{1-\kappa}$, under which profit is maximized at $p_{2,2}$.
4. $p_{2,1}$ is an interior solution within $\left[\frac{D_1}{D_1+D_2}\delta(1-\kappa), p_2^{UB}\right]$, $p_{2,2} < p_{2,2}^{LB}$, and $\Pi_{2,1}(p_{2,1}) \leq \Pi_{2,2}\left(p_{2,2}^{LB}\right)$: Together with the initial requirement of $D_2 < q \leq D_1 + D_2$, the conditions for this case can be presented as $\max\left\{\frac{1}{2}(D_1+D_2)\left(1-\sqrt{\left(1-\frac{1}{(1-\kappa)}\left(1-\frac{D_1}{D_1+D_2}\kappa\right)^2\right)}\right), D_2\right\} \leq q \leq \frac{1}{2}(D_1+D_2)$ and $D_1(1-\kappa) \leq D_2 \leq D_1\sqrt{1-\kappa}$, under which profit is maximized at $p_{2,2}^{LB}$.

5. $p_{2,1}$ is an interior solution within $\left[\frac{D_1}{D_1+D_2}\delta\left(1-\kappa\right), p_2^{UB}\right]$, $p_{2,2} < p_{2,2}^{LB}$, and $\Pi_{2,1}\left(p_{2,1}\right) \geq \Pi_{2,2}\left(p_{2,2}^{LB}\right)$: Together with the initial requirement of $D_2 < q \leq D_1 + D_2$, the conditions for this case are $D_2 < \frac{1}{2}\left(D_1+D_2\right)\left(1-\sqrt{\left(1-\frac{1}{(1-\kappa)}\left(1-\frac{D_1}{D_1+D_2}\kappa\right)^2\right)}\right)$, $D_1\left(1-\kappa\right) \leq D_2 \leq D_1\sqrt{1-\kappa}$, and $D_2 \leq q < \frac{1}{2}\left(D_1+D_2\right)\left(1-\sqrt{\left(1-\frac{1}{(1-\kappa)}\left(1-\frac{D_1}{D_1+D_2}\kappa\right)^2\right)}\right)$, under which profit is maximized at $p_{2,1}$.
6. $p_{2,1} < \frac{D_1}{D_1+D_2}\delta\left(1-\kappa\right)$ and $p_{2,2}$ is an interior solution within $\left[p_2^{LB}, \frac{D_1}{D_1+D_2}\delta\left(1-\kappa\right)\right]$: Together with the initial requirement of $D_2 < q \leq D_1 + D_2$, the conditions for this case are $\frac{D_1+D_2}{2} \leq q \leq D_1 + D_2$ and $D_2 \leq D_1\left(1-\kappa\right)$, under which profit is maximized at $p_{2,2}$.
7. $p_{2,1} < \frac{D_1}{D_1+D_2}\delta\left(1-\kappa\right)$ and $p_{2,2} \leq p_2^{LB}$: Together with the initial requirement of $D_2 < q \leq D_1 + D_2$, the conditions for this case are $D_2 < q \leq \frac{D_1+D_2}{2}$ and $D_2 \leq D_1\left(1-\kappa\right)$, under which profit is maximized at $p_{2,2}^{LB}$.

Finally, combining both Case I and Case II, the optimal second period pricing can be presented as follows:

- If $D_2 \leq \left(1-\kappa\right) D_1$, then

$$p_2^*\left(D_1, D_2, q\right) = \begin{cases} \delta\left(1-\frac{q}{D_1+D_2}-\frac{D_1}{D_1+D_2}\kappa\right) & \text{, if } 0 < q \leq D_2; \\ \delta\left(1-\kappa\right)\left(1-\frac{q}{D_1+D_2}\right) & \text{, if } D_2 < q \leq \frac{D_1+D_2}{2}; \\ \frac{\delta}{2}\left(1-\kappa\right) & \text{, if } q > \frac{D_1+D_2}{2}. \end{cases}$$

- If $D_1\left(1-\kappa\right) < D_2 \leq D_1\sqrt{1-\kappa}$, then

$$p_2^*\left(D_1, D_2, q\right) = \begin{cases} \delta\left(1-\frac{q}{D_1+D_2}-\frac{D_1}{D_1+D_2}\kappa\right) & \text{, if } 0 < q \leq \frac{(1-\kappa)D_1+D_2}{2}; \\ \frac{\delta}{2}\left(1-\frac{D_1}{D_1+D_2}\kappa\right) & \text{, if } \frac{(1-\kappa)D_1+D_2}{2} < q \leq \bar{q}; \\ \delta\left(1-\kappa\right)\left(1-\frac{q}{D_1+D_2}\right) & \text{, if } \bar{q} < q \leq \frac{1}{2}\left(D_1+D_2\right); \\ \frac{\delta}{2}\left(1-\kappa\right) & \text{, if } q > \frac{D_1+D_2}{2}. \end{cases}$$

  where $\bar{q} = \max\left\{\frac{1}{2}\left(D_1+D_2\right)\left(1-\sqrt{\left(1-\frac{1}{(1-\kappa)}\left(1-\frac{D_1}{D_1+D_2}\kappa\right)^2\right)}\right), D_2\right\}$.
- If $D_2 > D_1\sqrt{1-\kappa}$, then

$$p_2^*\left(D_1, D_2, q\right) = \begin{cases} \delta\left(1-\frac{q}{D_1+D_2}-\frac{D_1}{D_1+D_2}\kappa\right) & \text{, if } 0 < q \leq \frac{(1-\kappa)D_1+D_2}{2}; \\ \frac{\delta}{2}\left(1-\frac{D_1}{D_1+D_2}\kappa\right) & \text{, if } q > \frac{(1-\kappa)D_1+D_2}{2}. \end{cases}$$

Proof of Proposition 6 To attract consumers to purchase in the first period, the seller need to set her first-period price no larger than p_1^{MP}, which is the highest price under which consumers are indifferent between an immediate purchase and a wait decision. In other words, p_1^{MP} solves the equation $S_P^{MP}\left(p_1^{MP}\right) = S_W^{MP}$ and can be explicitly expressed as in Equ (8). When all consumers purchase in the first period, the seller's revenue performance is maximized by charging her first-period price at p_1^{MP}. Accordingly, the seller's optimal revenue performance is given by the maximum between the case where all first-period consumers purchase, $\pi_{AP}^{MP}(Q)$, and the case where no consumer purchase in the first period, $\pi_2^{NP}(Q)$.

Proof of Proposition 7 Under the same price p_2, we define the induced demand with the marketplace as $D^{MP}(p_2) \doteq \left(1 - \left(p_2 - \left(\kappa p_2 - \frac{D_1}{D_1+D_2}\delta(1-\kappa)\kappa\right)^+\right)/(\delta(1-\kappa))\right)(D_1+D_2)$ and without the marketplace as $D^{NMP}(p_2) \doteq \left(\left(1-\frac{p_2}{\delta(1-\kappa)}\right)^+ D_1 + \left(1-\frac{p_2}{\delta}\right)D_2\right)$, see Section **??** and Appendix 1. Denote $\Delta(p_2)$ as the induced demand difference between with and without the marketplace, $\Delta(p_2) \doteq D^{MP}(p_2) - D^{NMP}(p_2)$. Note that $\Delta(p_2)$ is differentiable everywhere with respect to p_2 except at the points where $\kappa p_2 - \frac{D_1}{D_1+D_2}\delta(1-\kappa)\kappa = 0$ and $1 - \frac{p_2}{\delta(1-\kappa)} = 0$. It is direct to show that $d\Delta/dp_2$ is positive when $p_2 \in \left(\frac{D_1}{D_1+D_2}\delta(1-\kappa), \delta(1-\kappa)\right)$ and negative when $p_2 \in \left[0, \frac{D_1}{D_1+D_2}\delta(1-\kappa)\right)$ and $p_2 \in (\delta(1-\kappa), \delta)$. Further note that $\Delta(0) = \Delta(\delta(1-\kappa)) = 0$. Together with the factor that Δ is a continuous function on p_2, we have shown that $\Delta(p_2) \le 0$ for $p_2 \in [0, \delta)$. Therefore, the seller will generate more demand without the marketplace under the same second-period price than without the marketplace. This observation directly implies the second part of this proposition by noticing that the second-period revenue performance with and without the marketplace is given by $p_2 \cdot \min\{q, D^{MP}(p_2)\}$ and $p_2 \cdot \min\{q, D^{NMP}(p_2)\}$. At last, the third part of this proposition comes from simply comparing the second-period price in Proposition EC.1 of Appendix 1 and that in Theorem 2.

Proof of Theorem 3 Denote the lower bound for the support of the first-period market size as D_1^{LB}. If the initial inventory level Q is lower than D_1^{LB} and all consumers in the first-period decide to purchase, then the seller will sell out her inventory in the first period. If there is a marketplace, then the equilibrium market clearing-price for the used products will be $p_2^E \doteq \delta\kappa\left(1 - \frac{Q}{D_2+D_1}\right)$, according to Theorem 1. As there is no inventory left for the second period, the seller's second-period price can be set arbitrarily high, and the corresponding second-period revenue will be zero. Following the analysis in Section 4.4, we can identify the expected surplus from an immediate purchase and that for a wait decision as $S_P^{MP}(p_1) \doteq \mathrm{E}\left[\frac{Q}{D_1}\cdot(v - p_1 + \max\{\delta\kappa v, p_2^E\})\right]$ and $S_W^{MP} \doteq \mathrm{E}\left[(\delta\kappa v - p_2^E)^+\right]$, respectively. Then, we can state the seller's revenue performance when all first-period consumers purchase in the first period (see Proposition 6) as

$$\pi_{AP}^{MP}(Q) = \mathrm{E}\left[Q\mathrm{E}\left[\frac{Q}{D_1}\left(v + \left(\delta\kappa v - p_2^E\right)^+ + p_2^E\right) - \left(\delta\kappa v - p_2^E\right)^+\right]/\mathrm{E}\left[\frac{Q}{D_1}\right]\right].$$

When there is no marketplace, the optimal revenue performance where all first-period consumers purchase (See Appendix 1) is $\pi_{AP}^{NM}(Q) = \frac{1}{2}Q$. We can simplify the revenue difference between with and without the marketplace as follows:

$$\pi_{AP}^{MP}(Q) - \pi_{AP}^{NM}(Q) = \mathrm{E}\left[\frac{1}{2}Q\kappa\frac{\delta}{D_1(D_1+D_2)^2}\left(Q^2 - 3QD_1 - 2QD_2 + 2D_1^2 + 4D_1D_2 + 2D_2^2\right)\right]/\mathrm{E}\left[\frac{Q}{D_1}\right],$$

which increases in κ and δ. As $\frac{d}{dQ}\left(Q^2-3QD_1-2QD_2+2D_1^2+4D_1D_2+2D_2^2\right)<0$, we can show that $Q^2-3QD_1-2QD_2+2D_1^2+4D_1D_2+2D_2^2\geq D_1^2-3D_1^2+2D_1^2+4D_1D_2+2D_2^2-2D_1D_2=2D_2^2+2D_1D_2>0$. Therefore, $\pi_{AP}^{MP}(Q)-\pi_{AP}^{NM}(Q)>0$ and immediately $\pi^{MP}(Q)\geq\pi^{NM}(Q)$. Hence, there must exist a therashold $Q^{LB}\geq D_1^{LB}$ such that when the initial inventory level Q is smaller than this threshold, we have $\pi^{MP}(Q)\geq\pi^{NM}(Q)$.

Proof of Theorem 4 We first prove the first part (a) of this proposition. Let the initial inventory level $Q\geq\frac{3D_1+D_2}{2}$ and the ratio of two-period market sizes $D_1/D_2<1$. We first consider the case where there exists a marketplace. From Theorem 2, we can show that the seller's optimal second-period price is $p_2^*(D_1,D_2,q)=\frac{\delta}{2}\left(1-\frac{D_1}{D_1+D_2}\kappa\right)$ and the second-period revenue is $\pi_2(D_1,D_2,q)=\frac{1}{4}\frac{\delta}{D_1+D_2}\left(D_1+D_2-\kappa D_1\right)^2$. As the supply of used products will never exceed the demand in the second period, from Theorem 1, the equilibrium market price is $p_2^E=\frac{1}{2}\kappa\frac{\delta}{D_1+D_2}\left(D_2-(1-\kappa)D_1\right)$. Following similar analysis as in Section 4.4, we first obtain this consumer's second-period incremental surplus

$$S_2^P(v)=\begin{cases}\frac{1}{2}\delta\frac{\kappa-1}{D_1+D_2}\left(D_1+D_2-2vD_1-2vD_2+\kappa D_1\right) & \text{, if } D_1<Q \text{ and } v\geq\frac{1}{2(D_1+D_2)}\left(D_2+D_1+\kappa D_1\right);\\ -\frac{1}{2}\kappa\frac{\delta}{D_1+D_2}\left(D_1-D_2+2vD_1+2vD_2-\kappa D_1\right) & \text{, if } D_1<Q \text{ and } v<\frac{1}{2(D_1+D_2)}\left(D_2-D_1+\kappa D_1\right);\\ 0 & \text{, otherwise.}\end{cases}$$

Then, using algebra, we can simplify the surplus from an immediate purchase to be

$$S_P^{MP}(p_1)=\frac{1}{8\left(D_1+D_2\right)^2}\begin{pmatrix}4D_1^2+4D_2^2+4D_1D_2-4\kappa D_1^2+\delta D_1^2+\delta D_2^2+2\kappa\delta D_1^2\\+4\kappa\delta D_2^2-3\kappa^2\delta D_1^2+2\delta D_1D_2+4\kappa^2\delta D_1D_2-2\kappa\delta D_1D_2\end{pmatrix},$$

and the surplus from a wait decision to be

$$S_W^{MP}=\frac{1}{8\left(D_1+D_2\right)^2}\left(-4\delta\kappa^3D_1^2+9\delta\kappa^2D_1^2-6\delta\kappa D_1^2+2\delta\kappa D_1D_2+5\delta D_1^2+2\delta D_1D_2+\delta D_2^2\right).$$

Immediately following Proposition 6, we have the optimal first-period price when all consumers purchase in the first period to be

$$p_1^{MP}=\frac{1}{2\left(D_1+D_2\right)^2}\begin{pmatrix}D_1^2+D_2^2+D_1D_2-\kappa D_1^2-\delta D_1^2+2\kappa\delta D_1^2\\+\kappa\delta D_2^2-3\kappa^2\delta D_1^2+\kappa^3\delta D_1^2+\kappa^2\delta D_1D_2-\kappa\delta D_1D_2\end{pmatrix},$$

under which the overall revenue performance is $\pi_{AP}^{MP}(Q)=\mathrm{E}[p_1^{MP}D_1+\pi_2(D_1,D_2,q)]$.

Now, consider the case where there is no marketplace. From Appendix 1, we can show that the seller's optimal second-period price is $p_2^{NM}(D_1,D_2,q)=\frac{\delta(1-\kappa)}{2}\frac{D_1+D_2}{D_1+(1-\kappa)D_2}$, if $\kappa\leq\frac{1}{2}$; otherwise, $p_2^{NM}(D_1,D_2,q)=\frac{\delta}{2}$.

If $\kappa\leq\frac{1}{2}$, then the seller's second-period revenue will be $\pi_2^{NM}(D_1,D_2,q)=\delta(1-\kappa)\frac{(D_1+D_2)^2}{4D_1+4D_2-4\kappa D_2}$. Through algebra, we can simplify the optimal first-period price under the case when all consumers purchase in the first period to be $p_1^{NM}=\frac{1}{8}\left(D_1+D_2\right)^2\frac{-\delta\kappa^2+\delta\kappa+1}{(D_1+D_2-\kappa D_2)^2}$ and the corresponding overall revenue to be

$$\pi_{AP}^{NM}(Q)=\frac{1}{8}\frac{\left(D_1+D_2\right)^2}{\left(D_1+D_2-\kappa D_2\right)^2}\left(D_1+2\delta D_1+2\delta D_2-\kappa^2\delta D_1+2\kappa^2\delta D_2-\kappa\delta D_1-4\kappa\delta D_2\right).$$

It is direct to show that the sign of $\frac{8(D_1+D_2)^2(D_1+D_2-\kappa D_2)^2}{D_1^5}\left(\pi_1^{AP}(Q)-\pi_{AP}^{NM}(Q)\right)$ depends on the coefficient of the fourth order of D_2/D_1, $(4\kappa^2-8\kappa+\kappa\delta-\kappa^2\delta+3)$, which is positive for $\kappa\leq\frac{1}{2}$. Therefore, there exists a threshold on D_2/D_1: if D_2/D_1 is higher than this threshold, $(\pi_{AP}^{MP}(Q)-\pi_{AP1}^{NM}(Q))$ will be positive and increase in δ. Finally, the sign of the first-order derivative of $\frac{8(D_1+D_2)^2}{D_1^5}\left(\pi_{AP}^{MP}(Q)-\pi_{AP}^{NM}(Q)\right)$ depends on the coefficient of the fifth order of D_2/D_1, $(\kappa\delta-\delta+2)$, which is positive for $\kappa\leq\frac{1}{2}$, and therefore $(\pi_{AP}^{MP}(Q)-\pi_{AP}^{NM}(Q))$ increases in κ.

If $\kappa>\frac{1}{2}$, then the seller's second-period price is $p_2^{NM}(D_1,D_2,q)=\frac{\delta}{2}$ and the corresponding revenue will be $\pi_2^{NM}(D_1,D_2,q)=\frac{\delta}{4}D_2$. Through algebra, we can simplify the optimal first-period price to be $p_1^{NM}=\frac{1}{8}\kappa\delta+\frac{1}{2}$ and the overall revenue performance to be $\pi_{AP}^{NM}(Q)=\left(\frac{1}{8}\kappa\delta+\frac{1}{2}\right)D_1+\frac{\delta}{4}D_2$. It is direct to show that the sign of $(\pi_{AP}^{MP}(Q)-\pi_{AP}^{NM}(Q))$ depends on the coefficient of the second order of D_2/D_1, $\delta(2-\kappa)$, which is positive for $\frac{1}{2}<\kappa<1$. Therefore, there exists another threshold on D_2/D_1: if D_2/D_1 is higher than that threshold, $(\pi_{AP}^{MP}(Q)-\pi_{AP}^{NM}(Q))$ will be positive, increase in δ, and decrease in κ.

Finally, the first part of this theorem follows directly from the observation that when the initial inventory is large, $\pi^{MP}(Q)=\pi_{AP}^{MP}(Q)$ and $\pi^{NM}(Q)=\pi_{AP}^{NM}(Q)$. The proof of the second part of this theorem is similar to that of the first part and, therefore, omitted.

Proof of Proposition EC.5 This proposition directly comes from Proposition EC.3 and Proposition EC.4 in Appendix 2.

Proof of Theorem EC.1 This theorem comes from the observation that the main model we derived in §4 is a special case for the Buy-Back program in which $r=0$, see discussions in Appendix 2.

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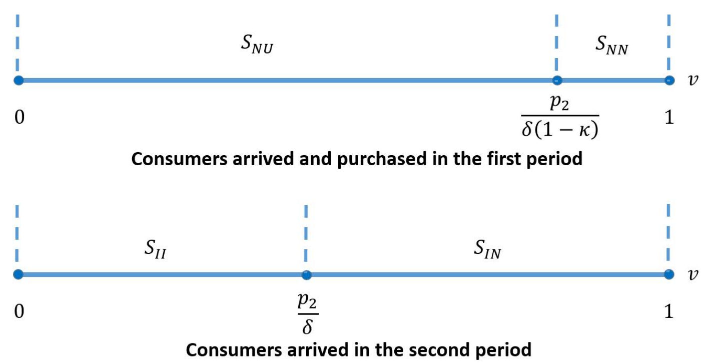
APPENDIX 1: The Benchmark Model (No Marketplace)

In this Appendix, we will present the analysis for the benchmark model where there is no marketplace for the used products. As in the main model, we start with the second-period problem. Similarly, we need to consider two cases, depending on whether consumers purchased in the first period. The seller's profit performance for the case where no consumer purchased in the first period has been analyzed in §4, and therefore we only need to discuss the case in which consumers purchased in the first period.

Without the marketplace, the second-period consumers will attempt to purchase a new unit if their surplus from doing so is positive: $\delta v > p_2$. Similarly, the first-period consumers will also attempt to purchase if purchasing new units generate higher surplus than holding the used units, i.e., $\delta v - p_2 > \delta\kappa v$. Figure EC.1 illustrates consumers' optimal decisions in the second period without the marketplace. The seller's second-period problem can be presented as follows:

Figure EC.1 **Consumers' optimal decisions in the second period without the marketplace.**



$$\pi_2^{NM}(D_1, D_2, q) \doteq \max_{0 \le p_2 \le \delta} \left\{ \left[p_2 \cdot \min\left\{ \left(1 - \frac{p_2}{\delta(1-\kappa)}\right)^+ D_1 + \left(1 - \frac{p_2}{\delta}\right) D_2, q \right\} \right] \right\}, \qquad \text{(EC.1)}$$

solving which we have the following proposition:

PROPOSITION EC.1. *If first-period consumers choose to purchase immediately in the first period, then in the second period, it is optimal for the seller to charge a price at*

$$p_2^{NM}(D_1,D_2,q)=\begin{cases}\frac{\delta(1-\kappa)}{2}\frac{D_1+D_2}{D_1+(1-\kappa)D_2} & \begin{array}{l}, \text{ if } q>\max\left\{\kappa D_2,\frac{D_1+D_2}{2}\right\},\ \kappa\le\frac{1}{2};\\ \text{or if } q>\max\left\{\kappa D_2,\frac{D_1+D_2}{2}\right\},\ D_1\ge\max\left\{D_2,\frac{2\kappa-1}{(1-\kappa)}D_2\right\},\ \kappa>\frac{1}{2};\\ \text{or if } q>\max\left\{\kappa D_2,\frac{D_1+D_2}{2}\right\},\ \frac{2\kappa-1}{(1-\kappa)}D_2<D_1<D_2,\ \frac{1}{2}<\kappa\le\frac{D_1+D_2}{2D_2};\end{array}\\ \frac{(D_1+D_2-q)\delta(1-\kappa)}{D_1+D_2(1-\kappa)} & \begin{array}{l}, \text{ if } \kappa D_2<q<\frac{D_1+D_2}{2},\ \kappa\le\frac{1}{2};\\ \text{or if } \max\{\tilde{q},\kappa D_2\}<q<\frac{D_1+D_2}{2},\ D_1\ge\max\left\{D_2,\frac{2\kappa-1}{(1-\kappa)}D_2\right\},\ \kappa>\frac{1}{2};\\ \text{or if } \max\{\tilde{q},\kappa D_2\}<q<\frac{D_1+D_2}{2},\ \frac{2\kappa-1}{(1-\kappa)}D_2\le D_1<D_2,\ \frac{1}{2}<\kappa\le\frac{D_1+D_2}{2D_2};\end{array}\\ \delta\left(\frac{D_2-q}{D_2}\right) & , \text{ if } q\le\min\left\{\kappa D_2,\frac{1}{2}D_2\right\};\\ \frac{\delta}{2} & , \text{ otherwise.}\end{cases}$$

where $\tilde{q}\doteq\left(\frac{(D_1+D_2)}{2}-\frac{1}{2}\sqrt{D_1\left(D_1+\frac{(1-2\kappa)D_2}{(1-\kappa)}\right)}\right)$.

Now, we analyze consumers' purchasing decisions. Consider an arbitrary consumer arrives at the first period, his expected surplus from an immediate purchase for this consumer is

$$S_P^{NM}(p_1)\doteq\mathrm{E}\left[\min\left\{1,\frac{Q}{D_1}\right\}\cdot\left(v-p_1+\max\left\{\delta\kappa v,\left(\delta v-p_2^{NM}\left(D_1,D_2,(Q-D_1)^+\right)\right)^+\right\}\right)\right],$$

and his expected surplus from a wait decision is

$$S_W^{NM}\doteq\mathrm{E}\left[A(D_1<Q)\cdot\left(\delta v-p_2^{NM}\left(D_1,D_2,(Q-D_1)^+\right)\right)^+\right].$$

Similar to the main model, we can show that the optimal price the seller should charge to attract consumers purchase in the first period is

$$p_1^{NM}=\frac{\mathrm{E}\left[\min\left\{1,\frac{Q}{D_1}\right\}\cdot\left(v+\max\left\{\delta\kappa v,\left(\delta v-p_2^{NM}\left(D_1,D_2,(Q-D_1)^+\right)\right)^+\right\}\right)\right]-S_W^{NM}}{\mathrm{E}\left[\min\left\{1,\frac{Q}{D_1}\right\}\right]},\quad\text{(EC.2)}$$

and the seller's optimal overall revenue performance can be characterized by the following proposition.

PROPOSITION EC.2. *When there is no marketplace, the seller's optimal revenue performance for the case where first-period consumers purchase immediately is*

$$\pi_{AP}^{NM}(Q)\doteq E\left[p_1^{NM}\cdot\min\{D_1,Q\}+\pi_2^{NM}\left(D_1,D_2,(Q-D_1)^+\right)\right].$$

Therefore, the seller's revenue performance $\pi^{NM}(Q)=\max\{\pi_{AP}^{NM}(Q),E[\pi_2^{NP}(D_1,D_2,Q)]\}$.

It is worth noting that similar to the marketplace case (Proposition 6), the strategy of setting a low first-period price to attract all consumers to purchase immediately (i.e., $\pi^{NM}(Q)$) does not necessarily dominate the strategy of waiting and selling to consumers only in the second period (i.e., $\mathrm{E}[\pi_2^{NP}(D_1,D_2,Q)]$).

Proof of Proposition EC.1 We consider two price segments: Segment I where $p_2 \leq \delta(1-\kappa)$ and Segment II where $p_2 > \delta(1-\kappa)$. In Segment I, the seller's revenue performance can be written as $p_2 \cdot \min\left\{\left(1-\frac{p_2}{\delta(1-\kappa)}\right) D_1 + \left(1-\frac{p_2}{\delta}\right) D_2, q\right\}$, whose unconstrained problem is a concave function and maximized at $p_{2,1} = \frac{\delta(1-\kappa)}{2}\frac{D_1+D_2}{D_1+(1-\kappa)D_2}$. Accordingly, the optimal second-period price for the Segment I is $\frac{\delta(1-\kappa)}{2}\frac{D_1+D_2}{D_1+(1-\kappa)D_2}$ for $\frac{\delta(1-\kappa)}{2}\frac{D_1+D_2}{D_1+(1-\kappa)D_2} < \delta(1-\kappa)$ and $\delta(1-\kappa)$ for $\frac{\delta(1-\kappa)}{2}\frac{D_1+D_2}{D_1+(1-\kappa)D_2} \geq \delta(1-\kappa)$. Similarly, in Segment II, the seller's revenue performance can be written as $p_2 \cdot \min\left\{\left(1-\frac{p_2}{\delta}\right) D_2, q\right\}$, whose unconstrained problem is a concave function and maximized at $p_{2,2} = \frac{\delta}{2}$. Therefore, the optimal second-period price for the Segment II is $\frac{\delta}{2}$ for $\frac{\delta}{2} > \delta(1-\kappa)$ or $\delta(1-\kappa)$ for $\frac{\delta}{2} \leq \delta(1-\kappa)$. Now, we need to include the leftover inventory constraint. If $q \leq \left(1-\frac{\delta(1-\kappa)}{\delta}\right) D_2$, or equivalently $q \leq \kappa D_2$, then it never optimal for the seller to charge price lower than $\delta(1-\kappa)$ (as doing so will not induce any additional sales). Therefore, under $q \leq \kappa D_2$, through simple algebra by comparing the leftover inventory and the demand induced by $p_{2,1}$, we can identify the the seller's optimal second-period price as

$$\tilde{p}_2 = \begin{cases} \frac{\delta}{2} & \text{, if } \frac{1}{2}D_2 \leq q \leq \kappa D_2,\ \kappa > \frac{1}{2}; \\ \left(1-\frac{q}{D_2}\right)\delta & \text{, if } q \leq \min\left\{\kappa D_2, \frac{1}{2}D_2\right\}. \end{cases}$$

Next we consider the case where $q > \kappa D_2$. Similarly, by comparing the leftover inventory and the revenue induced by $p_{2,1}$ and $p_{2,2}$, we can identify the seller's optimal second-period prices

$$\tilde{p}_2 = \begin{cases} & \text{, if } q > \max\left\{\kappa D_2, \frac{D_1+D_2}{2}\right\},\ \kappa \leq \frac{1}{2}; \\ \frac{\delta(1-\kappa)}{2}\frac{D_1+D_2}{D_1+(1-\kappa)D_2} & \text{or if } q > \max\left\{\kappa D_2, \frac{D_1+D_2}{2}\right\},\ D_1 \geq \max\left\{D_2, \frac{2\kappa-1}{(1-\kappa)}D_2\right\},\ \kappa > \frac{1}{2}; \\ & \text{or if } q > \max\left\{\kappa D_2, \frac{D_1+D_2}{2}\right\},\ \frac{2\kappa-1}{(1-\kappa)}D_2 < D_1 < D_2,\ \frac{1}{2} < \kappa \leq \frac{D_1+D_2}{2D_2}; \\ & \text{, if } \kappa D_2 < q < \frac{D_1+D_2}{2},\ \kappa \leq \frac{1}{2}; \\ \frac{(D_1+D_2-q)\delta(1-\kappa)}{D_1+D_2(1-\kappa)} & \text{or if } \max\{\tilde{q}, \kappa D_2\} < q < \frac{D_1+D_2}{2},\ D_1 \geq \max\left\{D_2, \frac{2\kappa-1}{(1-\kappa)}D_2\right\},\ \kappa > \frac{1}{2}; \\ & \text{or if } \max\{\tilde{q}, \kappa D_2\} < q < \frac{D_1+D_2}{2},\ \frac{2\kappa-1}{(1-\kappa)}D_2 \leq D_1 < D_2,\ \frac{1}{2} < \kappa \leq \frac{D_1+D_2}{2D_2}; \\ \frac{\delta}{2} & \text{, otherwise.} \end{cases}$$

Finally, summarize these two conditions, we can present the seller's optimal second-period price as in the proposition.

APPENDIX 2: The Buy-Back Program

We start with the seller's and consumers' optimal decisions in the second-period. Similarly to Section 4, we first consider the case when inventory is exhaust in the first period. Consumers possessing used units will contemplate among holding their used units, selling in the marketplace at price $\tilde{p}_2^E$, and selling back to the seller at price r. The following proposition characterize the equilibrium outcome among consumers in the marketplace.

PROPOSITION EC.3. *If all new products are sold to consumers who arrived in the first period, then it is optimal for the seller to set her second-period price at δ. Under this scenario, the seller's second-period revenue is* $\tilde{\pi}_2(D_1, D_2, Q, r) = -(r-s)\left(Q - (D_1+D_2)\cdot\left(1-\frac{r}{\delta\kappa}\right)^+\right)$

if $r > \delta\kappa\left(1-\frac{Q}{D_2+D_1}\right)$*; or* 0 *otherwise. In the marketplace, the equilibrium price is* $\tilde{p}_2^E = \max\left\{r, \delta\kappa\left(1-\frac{Q}{D_2+D_1}\right)\right\}$*, and consumers will* attempt *to follow the optimal decision rules described Proposition 2.*

Similarly, the following proposition establishes consumers optimal decisions under the Buy-Back program for the case where first-period consumers purchased and there is leftover inventory for the second period.

Proposition EC.4. *If first-period consumers purchased immediately upon arrival and there is leftover inventory for the second period, then for a given second-period price* p_2*, the equilibrium price in the marketplace is*

$$\tilde{p}_2^E = \max\left\{r, \left(\kappa p_2 - \frac{D_1}{D_1+D_2}\delta\left(1-\kappa\right)\kappa\right)^+\right\}.$$

Consumers will follow the optimal decision rules described in Proposition 3 and Proposition 4.

Combining these two propositions, the following proposition summarizes the equilibrium outcome among consumers in the marketplace.

Proposition EC.5. *Under the Buy-Back Program with a Buy-Back price* r*, the equilibrium market-clearing price in the marketplace is* $\tilde{p}_2^E = \max\{r, p_2^E\}$*, where* p_2^E *is defined in Proposition 2 and Theorem 1. Consumers in the second period will follow the optimal decision rules described in Proposition 3 and Proposition 4.*

To identify the seller's optimal second-period pricing decision, we need to consider three cases for a given Buy-Back price r: Case 1: $r \leq \left(\kappa p_2 - \frac{D_1}{D_1+D_2}\delta\left(1-\kappa\right)\kappa\right)^+$; Case 2: $\left(\kappa p_2 - \frac{D_1}{D_1+D_2}\delta\left(1-\kappa\right)\kappa\right)^+ < r \leq \kappa p_2$; Case 3: $p_2 \geq r > \kappa p_2$. Note that in the first case where $r \leq \left(\kappa p_2 - \frac{D_1}{D_1+D_2}\delta\left(1-\kappa\right)\kappa\right)^+$, both consumers' and the seller's decisions will be identical to the case without the Buy-Back program. Specifically, in Case 1, the seller's second-period and first-period problems are identical to that in Section 4. We therefore only need to consider the other two cases.

In Case 2, used products will be sold first to satisfy the demand in the marketplace and then sold back to the seller. The following proposition characterizes the seller's optimal second-period price.

Proposition EC.6. *For a given Buy-Back price* $r \in \left(\left(\kappa p_2 - \frac{D_1}{D_1+D_2}\delta(1-\kappa)\kappa\right)^+, \kappa p_2\right]$ *and a leftover inventory level* $q > 0$*, it is optimal for the seller to charge a price according to the following scheme:*

$$\tilde{p}^*_{2,2} = \begin{cases} r/\kappa & , \text{ if } \max\left\{\delta\kappa\left(1-\frac{q}{D_1+D_2}\right), \frac{1}{2}\delta\kappa - \frac{1}{2}s\frac{\kappa}{1-\kappa}\right\} < r \le \delta\kappa; \\ r - \frac{1}{2}s + \frac{1}{2}\delta(1-\kappa) & , \text{ if } \delta\kappa\left(1-\frac{q}{D_1+D_2}\right) < r \le \min\left\{\delta\kappa, \frac{1}{2}\delta\kappa - \frac{1}{2}s\frac{\kappa}{1-\kappa}\right\}; \\ r - \frac{1}{2}s + \frac{1}{2}\delta(1-\kappa) & , \text{ if } r \le \delta\kappa\left(1-\frac{q}{D_1+D_2}\right) \text{ and } q \ge \frac{(D_1+D_2)}{2}\left(1+\frac{s}{\delta(1-\kappa)}\right); \\ \delta(1-\kappa)\left(1-\frac{q}{D_1+D_2}\right)+r & , \text{ if } r \le \delta\kappa\left(1-\frac{q}{D_1+D_2}\right) \text{ and } q < \frac{(D_1+D_2)}{2}\left(1+\frac{s}{\delta(1-\kappa)}\right); \end{cases}$$

and the corresponding profit is given by

$$\tilde{\pi}_{2,2} = p^*_{2,2}\left(1-\frac{p^*_{2,2}-r}{\delta(1-\kappa)}\right)(D_1+D_2) - (r-s)\left(\left(1-\frac{p^*_{2,2}-r}{\delta(1-\kappa)}+\frac{r}{\kappa\delta}\right)D_1 - \left(\frac{p^*_{2,2}-r}{\delta(1-\kappa)}-\frac{r}{\kappa\delta}\right)D_2\right).$$

In Case 3, all used products will be sold back to the seller. The following proposition characterizes the seller's optimal second-period price.

Proposition EC.7. *For a given Buy-Back price* $p_2 \ge r > \kappa p_2$ *and a leftover inventory level* $q > 0$*, it is optimal for the seller to charge a price according to the following scheme:*

$$\tilde{p}^*_{2,3} == \begin{cases} \frac{1}{2}\delta & , \text{ if } q \ge \frac{1}{\delta}\left(\delta - \frac{r}{\kappa}\right)(D_1+D_2), \ \frac{1}{2}\delta\kappa \le r < \delta\left(1-\frac{q}{D_1+D_2}\right), \text{ and } q \ge \frac{D_1+D_2}{2}; \\ \frac{1}{2}\delta & , \text{ if } q \ge \frac{1}{\delta}\left(\delta - \frac{r}{\kappa}\right)(D_1+D_2), \text{ and } \max\left\{\delta\left(1-\frac{q}{D_1+D_2}\right), \frac{1}{2}\delta\kappa\right\} \le r < \frac{\delta}{2}; \\ r & , \text{ if } q \ge \frac{1}{\delta}\left(\delta - \frac{r}{\kappa}\right)(D_1+D_2), \text{ and } \max\left\{\delta\left(1-\frac{q}{D_1+D_2}\right), \frac{\delta}{2}\right\} \le r \le \delta; \\ r/\kappa & , \text{ if } q \ge \frac{1}{\delta}\left(\delta - \frac{r}{\kappa}\right)(D_1+D_2), \ r \le \min\left\{\delta\left(1-\frac{q}{D_1+D_2}\right), \frac{1}{2}\delta\kappa\right\}, \text{ and } q \ge \frac{D_1+D_2}{2}; \\ r/\kappa & , \text{ if } q \ge \frac{1}{\delta}\left(\delta - \frac{r}{\kappa}\right)(D_1+D_2), \text{ and } \delta\left(1-\frac{q}{D_1+D_2}\right) \le r \le \frac{1}{2}\kappa\delta; \\ \delta\left(1-\frac{q}{D_1+D_2}\right) & , \text{ if } q \ge \frac{1}{\delta}\left(\delta - \frac{r}{\kappa}\right)(D_1+D_2), \ r \le \delta\left(1-\frac{q}{D_1+D_2}\right), \text{ and } q < \frac{D_1+D_2}{2}; \end{cases}$$

and the corresponding profit is given by

$$\tilde{\pi}_{2,3} = p^*_{2,3}\left(1-\frac{p^*_{2,3}}{\delta}\right)(D_1+D_2) - (r-s)(D_1).$$

For each given Buy-Back price r, we need to identify the candidates for the optimal second-period price by verifying the price for these three cases satisfy their initial conditions respectively. The rest of analysis is similar to that of Section 4.4 and therefore omitted. Finally, it is straightforward to show that the Buy-Back program benefits the seller:

Theorem EC.1. *The Buy-Back program improves the seller's revenue performance, i.e.,* $\pi^{BB}(Q) \ge \pi^{MP}(Q)$.

Proof of Proposition EC.3 Clearly, if $r \le \delta\kappa\left(1-\frac{Q}{D_2+D_1}\right)$, then no consumers will sell their used products to the seller (as the marketplaces is a more attractive option). Therefore, consumers' behavior will be captured by Proposition 2. On the other hand, when $r > \delta\kappa\left(1 - \frac{Q}{D_2+D_1}\right)$, the equilibrium price will be equal to the Buy-Back price in order to attract consumers to sell in the marketplaces. Consumers will attempt to purchase (or sell) the used units if $\delta\kappa v - r \geq$ (or $<$)0, and the seller will need to absorb the additional units, $\left(\left(\frac{r}{\delta\kappa}\right)Q - \left(1 - \frac{r}{\delta\kappa}\right)(D_1 - Q + D_2)\right)$, which can not be sold to the marketplaces.

Proof of Proposition EC.4 Clearly, if $r \leq \left(\kappa p_2 - \frac{D_1}{D_1+D_2}\delta(1-\kappa)\kappa\right)^+$, then it is profitable for all consumers to resell their products in the marketplaces (Theorem 1). When $r > \left(\kappa p_2 - \frac{D_1}{D_1+D_2}\delta(1-\kappa)\kappa\right)^+$, the equilibrium market price will equal to r (otherwise, no consumer will be willing to sell in the marketplaces). Consumers optimal decisions under this case are captured by Proposition 3 and Proposition 4.

Proof of Proposition EC.5 This proposition directly comes from Proposition EC.3 and Proposition EC.4.

Proof of Proposition EC.6 The proof of this proposition is similar to Proposition 5 and Theorem 2 and therefore omitted.

Proof of Proposition EC.7 The proof of this proposition is similar to Proposition 5 and Theorem 2 and therefore omitted.

Proof of Theorem EC.1 This theorem comes from the observation that the main model we derived in §4 is a special case for the Buy-Back program in which $r = 0$.